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Analyses of Elastically Scattered Charged Pions from ^{40}Ca At 50mev

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□ ABSTRACT □

A simple local potential, extracted originally from available phase shifts using an inverse scattering theory with Klein-Gordon equation, was parameterized to describe the elastic scattering of charged pions from ^{40}Ca at 50 MeV. This will be a stringent test for the correctness of the simple local potential form. The Coulomb effects are simply considered by the fact that the π^- interaction with nucleons in the nucleus is at an effectively higher energy than the π^+. This procedure approximately is equivalent to other complicated theoretical treatments. The good agreement with the experimental data assures the validity of this approach and the success of our potential. This also provides a strong motivation to study the scattering of charged pions at other energies and for other nuclei where experimental data are available.

Key Words: Elastic Scattering, Charged Pions, Inverse Scattering Theory, Klein-Gordon Equation
PACS:

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تحليل التشتت المرن للبيونات المشحونة عن نواة الكالسيوم 40 على 50 مليون الكترون فولط

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□ الملخّص □

تمّ استخدام الجهد الموضعي البسيط المشتق أصلاً من الإزاحات الطورية باستخدام نظرية التشتت العكسي ومعادلة كلاين – جوردن وذلك بتغيير معاملاته لوصف التصادم المرن للبيونات المشحونة عن الكالسيوم 40 عند طاقة 50 مليون إلكترون فولط. هذه الدراسة ستؤكد صحة الجهد الموضعي البسيط (والذي تمّ اقتراحه وتطويره من قبلنا).

تمّ اعتبار التأثيرات الكولومية ببساطة بناءً على حقيقة أنّ البيونات السالبة تتفاعل مع نيوكليونات النواة بطاقة فعّالة أكبر من البيونات الموجبة. تكافئ هذه الطريقة تقريباً المعالجات النظرية المعقّدة.

يؤكد التوافق الحسن مع القراءات التجريبية صلاحية هذه الطريقة ونجاح هذا الجهد. وهذا يعطي أيضاً دفعاً قوياً لدراسة تشتت البيونات المشحونة على طاقات أخري وعن نويات أخرى حيثما تتوفر النتائج التجريبية.

Introduction

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Since the establishment of meson facilities in the 70's, a wealth of pion-nucleus data has been accumulated, and investigated, by experimentalists [1-5]. This enforces a great challenge on theoretician to explain these available data. To facilitate such studies, the pion-nucleus reactions have been classified into three major regions :

1) the low energy region : $0 \prec T_\pi \leq 100\,\text{MeV}$
2) the delta resonance region : $100\,\text{MeV} \prec T_\pi \leq 400\ \text{MeV}$
3) the high energy region : $400\,\text{MeV} \prec T_\pi$

Since the energy region between 50 and 100 MeV is of particular interest [6], one may subdivide the low energy region into two sub-regions:

a) the low energy region : $0 \prec T_\pi \prec 50\,\text{MeV}$
b) the intermediate (the transition)-energy region : $50\,\text{MeV} \leq T_\pi \leq 100\,\text{MeV}$

The elastic scattering of intermediate-energy charged pions from nuclei in the intermediate energy region is of special importance for many physicists. It is well known that the elastic scattering is a preliminary step for addressing the physics of other complex processes as inelastic scattering, charge exchange, double charge exchange, pion production, pion production and other reactions [7].

The intermediate-energy region is very important because it represents a bridge between the low energy region, with connections to the pionic-atom problem, and the delta resonance region. Also, and in this energy region, pions penetrate deeply into the nuclear interior because of the pion's long mean free path [8].

The elastic scattering of charged pions by different nuclei has been investigated by many workers [9]. It is a very rich subject for studying the effect of all Coulomb contributions, i.e. Coulomb and Coulomb-nuclear terms with necessary corrections. Theoretically, this is a very difficult subject and it will be approached and treated separately.

Different, non-local and local, potentials have been introduced and used to explain the pion-nucleus elastic scattering data in the different energy regions [10]. Most of these potentials are phenomenological and are difficult, if not impossible, to interpret in physical terms [11]. The most successful optical potential, simple and local, was introduced recently by Satchler [12]. Local potentials are easily modified to treat relativistic as well as non-relativistic scattering problems. Such an advantage of Satchler's treatment was grasped and used by other authors [13-14].

Here I am going to use a similar local optical potential, for pion-^{40}Ca case, whose analytical form was extracted from available phase shifts using Inverse Scattering Theory (IST) [15-16]. This potential proves its success in describing the π^+- ^{40}Ca angular data in the low –energy and delta resonance regions [17-18]. This creates a strong motivation to test the validity of the same potential form in describing the elastic scattering data of charged pions from ^{40}Ca at an intermediate-energy incident pion, namely $T_\pi = 50$ MeV.

At this stage, Stricker's treatment is followed [19]. The strategy of this treatment is based on the following : When a negative pion approaches the nucleus it is accelerated by the Coulomb field; a positive pion is decelerated. To account this, for the pion-^{40}Ca, Stricker suggested that the kinetic energy of the incident pion should be decreased by 7.6 MeV for positive pions and increased by 7.6 MeV for negative pions. Accordingly, this treatment requires no recast in our equations.

This paper includes another three sections. The methodology and material, including inverse scattering theory (IST) and extracted pion-^{40}Ca potential, is described in section 2.

Section 3 contains the results and their discussions. Section 4 summarizes some conclusions and recommendations.

Methodology and Material :

It is well-known that pion-nucleus interaction is governed by Klein-Gordon equation which can be written as :

$$\left[d^2/dr^2 + k^2 - U(r) - \frac{l(l+1)}{r^2}\right]R_{nl}(r) = 0 \qquad (1)$$

where $R_{nl}(r)$ is the r times the radial part of the wave function for a spherical symmetrical external potential. Also, in (1), k^2 and $U(r)$ are given by

$$k^2 = \left(E^2 - m^2c^4\right)/\hbar^2c^2 \qquad (2)$$

$$U(r) = \frac{2E}{\hbar^2c^2}\left[V(r) - V^2(r)/2E\right] = V_{eff} \qquad (3)$$

In (3), E and m are, respectively, total energy and pion rest mass and c is the velocity of electro-magnetic wave in vacuum. $V(r)$ is the complex pion-nucleus potential.

Since $V(r)$ is complex, the real and imaginary part of the effective potential, ReU and ImU, respectively, have the following expressions:

$$\mathrm{Re}U(r) = \left(\frac{2E}{\hbar^2c^2}\right)\left[\mathrm{Re}V(r) - \frac{\left\{(\mathrm{Re}V(r))^2 - (\mathrm{Im}V(r))^2\right\}}{2E}\right] \qquad (4)$$

$$\mathrm{Im}U(r) = \left(\frac{2E}{\hbar^2c^2}\right)\left[\mathrm{Im}V(r) - \frac{2(\mathrm{Re}V(r))(\mathrm{Im}V(r))}{2E}\right] \qquad (5)$$

For completeness and importance, the Inverse Scattering Theory (IST) used to extract the real and imaginary parts of the effective potential [20], is summarized here again. Introducing

$$\varphi_{nl}(r) = (kr)^{-(l+1)}R_{nl}(r) \qquad (6)$$

one obtains the following equation for $\varphi_{nl}(r)$ from (1)

$$\left[\frac{d^2}{dr^2} + \frac{2(l+1)}{r}\frac{d}{dr} + k^2 - U(r)\right]\varphi_{nl}(r) = 0 \qquad (7)$$

Dividing the range, R, of $U(r)$ in N equal parts, one has $R=\Delta N$ and the point $r=n\Delta$ with n being an integer. Replacing the differential operators by central differences, one may obtain from (7) the following difference equation

$$\varphi_{n+1} = A_n(l)B_n(l)\varphi_n + C_n(l)\varphi_{n-1};\quad n = 1,2...N \qquad (8)$$

In (8), we have suppressed the suffice (nl) in φ and $A_n(l)$, $B_n(l)$ and $C_n(l)$ are given by the following expressions

$$A_n(l) = 2 - \Delta^2k^2 + \Delta^2U_n \qquad (9)$$

$$B_n(l) = n/(l+1+n) \qquad (10)$$

$$C_n(l) = (l+1-n)/(l+1+n) \qquad (11)$$

and U_n is the value of U at the n-th point. The logarithmic derivative relevant for the calculation of phase shifts for a given l, $Z_N(l)$ is given by replacing the first derivative by central difference at $R=N\Delta$, and is the following:

$$Z_N(l) = \left(\frac{N}{2}\right)\left(\frac{\varphi_{N+1} - \varphi_{N-1}}{\varphi_N}\right) \tag{12}$$

The evaluation of (12) requires the knowledge of φ_N, φ_{N+1} and φ_{N-1} at $n=N$. For $n=N$, and (8) can be reduced to :

$$\frac{\varphi_{N+1}}{\varphi_N} = A_N(l)B_N(l) + C_N(l)/(\varphi_N / \varphi_{N-1}) \tag{13}$$

One may now replace $(\varphi_N / \varphi_{N-1})$ successively and obtain the following continued fraction equation

$$\frac{\varphi_{N+1}}{\varphi_N} = A_N(l)B_N(l) + \left[\frac{C_N(l)}{A_{N-1}(l)B_{N-1}(l)+}\frac{C_{N-1}(l)}{A_{N-2}(l)B_{N-2}(l)+}\cdots\frac{C_3(l)}{A_2(l)B_2(l)+}\frac{C_2(l)}{A_1(l)B_1(l)+C_1(l)/(\varphi_1/\varphi_0)}\right] \tag{14}$$

Since for a given l, there is always a point "m" where $C_m(l)=0$, the last term in the continued fraction does not enter in the calculation. One can similarly calculate the values of φ_{N-1}/φ_N to obtain $Z_N(l)$ using equation (12) and hence, the phase shift.

For the inverse scattering process, one starts at a point where $U_N=0$. At that point $A_N=2-\Delta^2k^2$, is known. Using equations (12) and (13) at that point one can get :

$$\frac{\varphi_N}{\varphi_{N-1}} = \frac{C_N(l)}{\left[\frac{2}{N}Z_N(l) - A_N(l)B_N(l)\right]} \tag{15}$$

where $l=0,1,2....L$ and $N=L+1$, L being the largest partial wave. As noted earlier, there is always an l_n that makes $C_n(l_{N-n})=o$. For $n=N-1$, we therefore, have

$$A_{N-1}(l_1) = \frac{1}{B_{N-1}(l_1)}\frac{\varphi_N(l_1)}{\varphi_{N-1}(l_1)}$$

(16)

This inward iteration may be continued to find all A_{N-j} at the points for $j=2,3...N-1$

$$A_{N-j}(l_j) = \frac{1}{B_{N-j}(l_j)}\left[\frac{C_{N+1-j}(l_j)}{-A_{N+1-j}(l_j)B_{N+1-j}(l_j)+}\cdots\frac{C_{N-2}(l_j)}{-A_{N-2}(l_j)B_{N-2}(l_j)+}\cdot\frac{C_{N-1}(l_j)}{-A_{N-1}(l_j)B_{N-1}(l_j)+\varphi_N(l_j)/\varphi_{N-1}(l_j)}\right]$$

(17)

Once $A_{N-j}(l_j)$ is known, then U_n can be obtained by the following equation :

$$U_n = \frac{1}{\Delta^2}\left[A_n - 2 + \Delta^2k^2\right] \tag{18}$$

Therefore, both the real and imaginary part of V_{eff} could be calculated.

Results and Discussion

Different forms of the optical potential have been used to study the pion-nucleus interactions. Although all these potential models gave a reasonable description of the data, they didn't account for diffraction minima with, in some cases, a discrepancy in magnitude and shape of the cross section [21]. Other optical potential models, with improvements, were also used but with a marginal success [22].

The potential used by Satchler, $V_s(r)$, has the following functional form :

$$V_s(r) = \frac{V_o}{1+\exp\left(\frac{r-R_o}{a_o}\right)} + i\frac{W_2}{1+\exp\left(\frac{r-R_2}{a_2}\right)} + i\frac{W_3\exp\left(\frac{r-R_3}{a_3}\right)}{\left[1+\exp\left(\frac{r-R_3}{a_3}\right)\right]^2} \quad (19)$$

As stated in my previous study [23], it was necessary to modify the real part of the Satchler's potential by adding to it the following term:

$$V_1(r) = \frac{V_1}{\left[1+\exp\left(\frac{r-R_1}{a_1}\right)\right]^2} \quad (20)$$

Thus, the new complex potential is :

$$V(r) = V_S(r) + V_1(r) \quad (21)$$

Here I am extending the use of the same potential form in equ. (21) but for intermediate- energy incident charged pions, namely T_π = 50 MeV, following both Satchler's treatment and Stricker's assertion.

The preliminary results of the calculated elastic scattering cross sections for charged pions scattered from ^{40}Ca at 50 MeV (solid line for positive pions; dashed line for negative pions) compared with the experimental data [24-25] (solid dots for positive pions; empty triangles for negative pions) are shown in figure (1). The agreements are reasonably good. It is worth noting that the potential parameters, shown in table (1), are the same for both charged pions.

One may also consider the reaction cross section as an additional constraint. Our calculated cross sections are 345.8 mb and 426.3 mb for positive and negative pions respectively. The value 345.8 mb is compared well with the published values [26]. Unfortunately, there is no published reaction cross section for the negative ones.

O. Meirav et al. [27] confirmed that a stringent test of the potential is its ability to account for angular distributions and to predict the cross sections.

Table (1): The potential parameters V_o (in MeV), R_o (in fm), a_o (in fm), V_1 (in MeV), R_1 (in fm), a_1 (in fm), W_2 (in MeV), R_2 (in fm), a_2 (in fm), W_3 (in MeV), R_3 (in fm), and a_3 (in fm), used in equation (21) for the 50 MeV incident charged pions (T_π in MeV).

T_π	V_o	R_o	a_o	V_1	R_1	a_1	W_2	R_2	a_2	W_3	R_3	a_3
50	33.5	3.569	0.653	580	1.45	0.183	90.0	1.551	0.450	111.6	2.187	0.45 0

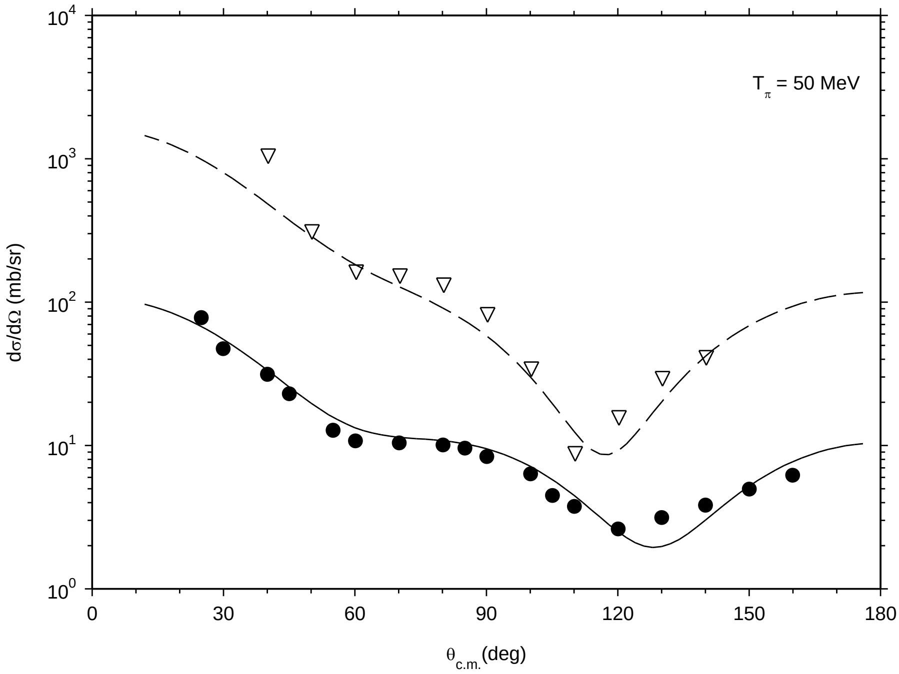

Fig. (1): The calculated angular distributions (solid line and dashed line) using the effective potential given by the equation (21) are compared with the experimental data[24-25] (solid circles and empty triangles) as a function of center of mass angle ($\theta_{c.m.}$) for T_π = 50 MeV (positive and negative pions) respectively.

Conclusions and Recommendations

The simple local potential form, extracted originally from the phase shifts using IST, proves to be successful in explaining the data at 50 MeV. The data includes both positive and negative pions scattered elastically from ^{40}Ca, which brings Coulomb effects into consideration. As a consequence, the potential parameters have to be changed to describe the 50 MeV measurements nicely.

As a preliminary study, and using our potential, it is verified that all Coulomb effects could be accounted for by a change in the effective energy of the incident pion. It is hoped, and recommended, that the validity of this potential continues to other nearby energies; namely for T_π = 65 and 80 MeV, and to the neighboring two energy regions, where experimental phase shifts and angular distributions are available, without major complexities. A comprehensive study for the elastic scattering of charged pions from different nuclei, and in all three energy regions, must be considered.

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