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Propagation of longitudinal waves in biological media

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□ ABSTRACT □

The main goal of the presented research in this paper is to find a general way to solve longitudinal vibration problems. This way must solve these problems in nonlinear elastic bar systems with a biological factor.

We applied longitudinal vibration equations in a nonlinear elastic bar with biological factor, the bar material was taken non-linear. and solve the problem in a bar of finite length.

Keywords: Elastic, Nonlinear, Wave, Reaction, Biological, vibration.

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انتشار الأمواج الطولانية في الأوساط البيولوجية

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□ ملخّص □

إن الهدف من البحث المقدم في هذا العمل هو وضع طريقة عامة لحل مسائل الاهتزازات الطولانية، وحل هذه المسائل بالنسبة لمنظومات قضبانية من مواد مرنة لاخطية وذلك بوجود العامل البيولوجي.

لقد استخدمنا معادلات الاهتزازات الطولانية على قضيب من مادة مرنة لاخطية آخذين بالاعتبار وجود العامل البيولوجي، حيث أخذت مادة القضيب لاخطية. وتم حل المسألة من أجل قضيب منتهي الطول.

الكلمات المفتاحية : مرونة - لاخطية - موجة - رد فعل - بيولوجي - اهتزاز .

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Introduction:

Biological industry needs the study of body behavior and this in turn, needs making good modeling of biological tissues.

Existing modules reflect in some way the properties of biological tissues by using a biological factor. On the other side, the biological tissue, muscular or skeletal, has clear elastic and viscoelastic properties and non-linear relation between operating forces and deformations. Positive operations like propagation of waves in such media need a special study that provides the relations determining frequency and amplitude properties depending on physical and other parameters. This is the base of resolving problems when producing biological members.

The important of research and its objectives:

Studying systems vibration which formed of active biologic substances with no doubt the mast important subject in protecting from vibration problems.

The main aim of this study is clarifying the role of vibration operations according the study subject.

Research methods and materials:

using some approximations and Fourier Transformation we put a general solving way with a set of related differential equations. Solving numerically this set of equations shows the role of non linearity and biological factor in the studied medium.

Let be an elastic biological body [1]. Depending on the theory of biological body, the equation of motion in one-dimensional media is done by:

(1) $$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x}$$

Where $\sigma(t)$ is the tension, it takes the following form:

(2) $$(1-A)\frac{\partial \sigma}{\partial x} + A\tau \frac{\partial^2 \sigma}{\partial x \partial t} = \rho \frac{\partial^2 u}{\partial t^2}$$

Here $0 < A < 1$, $0 < \tau << 1$ and τ is a parameter representing the time relaxation of the substance and is very small compared to t.

Using non-linear **Rabotnov** law and [2] we obtain:

(3) $$\sigma = \frac{1}{1+k^*}\varphi(\varepsilon) = (1-\Gamma^*)\varphi(\varepsilon)$$

Here k^* is the climbing operator. $\Gamma^* = \int_0^t \Gamma(t-\tau) f(s) ds$ is the auxiliary operator helping the resolution of k^*, $\varphi(\varepsilon)$ is a linear function that describes the instantly deformation curve. Where ε is the deformation. So the equation (2) becomes as follows:

(4) $$(1-A)\frac{\partial}{\partial x}[(1-\Gamma^*)\varphi(\varepsilon)] + A\tau \frac{\partial^2}{\partial x \partial t}[(1-\Gamma^*)\varphi(\varepsilon)] = \rho \frac{\partial^2 u}{\partial t^2}$$

If we consider the kernel of Γ^* to be regular and $\Gamma(t-\tau) = 0$ then the equation (4) takes the following form:

(5) $$[(1-A)\frac{\partial^2 u}{\partial x^2} + A\tau \frac{\partial^3 u}{\partial x^2 \partial t}]\frac{\partial \varphi}{\partial \varepsilon} + A\tau \frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial x \partial t}\frac{\partial^2 \varphi}{\partial \varepsilon^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

And this is an equation of a one-dimensional deformation in a non-linear elastic biological medium [3].

Using (5) we'll study the problem of sinusoidal wave propagation in a thin bar of finite length where the material is nonlinear elastic and able to react to external operators, with the following boundary conditions:

(6) $$u(x,t) = u_0 \cos \omega t \qquad ; x = 0$$

(7) $$\frac{\partial u}{\partial x} = 0 \qquad ; x = l$$

and explaining the effect of small vibration [4] in the case.

Let $\varphi(\varepsilon)$ in (5) take the form [5]:

(8) $$\varphi(\varepsilon) = a \ln(1 + \lambda \varepsilon)$$

Where a, λ are constants, and:

$$\tilde{x} = \frac{x}{l}, \tilde{u} = \frac{u}{l}, \tilde{\tau} = \frac{\tau}{t_0}, \tilde{t} = \frac{t}{t_0}, \tilde{\varepsilon} = \varepsilon, \tilde{\omega} = \omega t_0, \tilde{a} = \frac{a t_0^2}{\rho l^2}$$

Then we find:

(9)

$$[(1-A)\frac{\partial^2 u}{\partial x^2} + A\tau \frac{\partial^3 u}{\partial x^2 \partial t}](1 + \lambda \frac{\partial u}{\partial x}) - A\tau\lambda \frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial x \partial t} = \frac{\partial^2 u}{\partial t^2}[1 + 2\lambda \frac{\partial u}{\partial x} + \lambda^2 (\frac{\partial u}{\partial x})^2]$$

Where $a\lambda = E$ is **Young** Elasticity modulus. To simplify, we neglect signals over unmeasured values.

Equation (9) contains two parameters whose values are small and unknown:

λ : Instant deformation curve non-linearity parameter.

τ : Relaxation time

As equation (2) is obtained to first-degree approximation for parameter τ [6] so equation (5) and unknown function $u(x,t)$ cannot exceed this degree of approximation. This in turn, restricts the degree of approximation of function (8) For this we put:

(10) $$\lambda \sim \tau^{\frac{1}{2}}$$

This allows making the following substitutions:

(11) $$\lambda \sim \tau^{\frac{1}{2}}, \tau, \lambda^2 \sim \tau$$

By developing $u(x,t)$ following to λ, τ we find:

(12) $$u(x,t) = \sum_{m,n} \lambda^m \tau^n u_{m,n}(x,t)$$

Considering (11) (m, n) takes the following values:

$$(0,0), (0,1), (1,0), (2,0)$$

By substituting (12) in (9) and comparing terms of similar degrees, a set of related equations is obtained. That permits to find the following functions:

$u_{0,0}(x,t), u_{0,1}(x,t), u_{1,0}(x,t), u_{2,0}(x,t)$

And this gives the general solution according to (12)

(13) $$(1-A)\frac{\partial^2 u_{0,0}}{\partial x^2}-\frac{\partial^2 u_{0,0}}{\partial t^2}=0$$

(14) $$(1-A)\frac{\partial^2 u_{0,1}}{\partial x^2}-\frac{\partial^2 u_{0,1}}{\partial t^2}=A\frac{\partial^3 u_{0,0}}{\partial x^2 \partial t}$$

(15) $$(1-A)\frac{\partial^2 u_{1,0}}{\partial x^2}-\frac{\partial^2 u_{1,0}}{\partial t^2}=2\frac{\partial^2 u_{0,0}}{\partial t^2}\frac{\partial u_{0,0}}{\partial x}-(1-A)\frac{\partial^2 u_{0,0}}{\partial x^2}\frac{\partial u_{0,0}}{\partial x}$$

(16)
$$(1-A)\frac{\partial^2 u_{2,0}}{\partial x^2}-\frac{\partial^2 u_{2,0}}{\partial t^2}=$$
$$\frac{\partial^2 u_{1,0}}{\partial t^2}\frac{\partial u_{0,0}}{\partial x}-(1-A)[\frac{\partial^2 u_{1,0}}{\partial x^2}\frac{\partial u_{0,0}}{\partial x}+\frac{\partial^2 u_{0,0}}{\partial x^2}\frac{\partial u_{1,0}}{\partial x}]+\frac{\partial^2 u_{0,0}}{\partial t^2}\frac{\partial u_{1,0}}{\partial x}+\frac{\partial^2 u_{0,0}}{\partial t^2}(\frac{\partial u_{0,0}}{\partial x})^2$$

Considering boundary conditions provided by (6) and (7) the solutions for (13) - (16) can be written as follows:

(17) $$u_{0,0}(x,t)=f_{0,0}(x)\cos\omega t$$
$$f_{0,0}(x)=\frac{u_0\cos[\frac{\omega(l-x)}{\sqrt{1-A}}]}{\cos(\frac{\omega l}{\sqrt{1-A}})}$$

(18) $$u_{0,1}(x,t)=f_{0,1}(x)\sin\omega t$$
$$f_{0,1}(x)=\frac{-A\omega^2 u_0}{2(1-A)}\frac{l\sin\frac{\omega x}{\sqrt{1-A}}-x\sin\frac{\omega(x-l)}{\sqrt{1-A}}\cos\frac{\omega l}{\sqrt{1-A}}}{\cos^2(\frac{\omega l}{\sqrt{1-A}})}$$

(19) $$u_{1,0}(x,t)=g_{1,0}(x)+f_{1,0}(x)\cos 2\omega t$$

(20) $$u_{2,0}(x,t)=g_{2,0}(x)\cos\omega t+f_{2,0}(x)\cos 3\omega t$$

Where functions $f_{1,0}(x), g_{1,0}(x), f_{2,0}(x), g_{2,0}(x)$ have complicated forms and therefore we wont write them now.

The solutions (17) and (20) using (12) determine the function $u(x,t)$ giving the position of the free end of the bar.

$$u(x,t)=u_{0,0}(x,t)+\tau u_{0,1}(x,t)+\lambda u_{1,0}(x,t)+\lambda^2 u_{2,0}(x,t)$$

Conclusion:

These solutions confirm that the existence of non-linearity implies the generation of sinusoids of high degrees.

By giving the unknown parameters the following values:

$$x=l=1, u_0=0.01, \omega=0.05, A=0.5, \tau=0.01, \lambda=\sqrt{\tau}$$

And making the calculations we prove that: Low influence of non-linearity and biological factor are proportionally correlated to the small values of the frequency ω and the amplitude u_0, and it increases whenever the values of these two parameters increase.

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