0Derivation of Hamiltonian and Symplectic Structure from Tellegen's Theorem as an Approach to the Quantum Quantization of Electrical Circuits

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ABSTRACT

In this research, we present a new theoretical framework that links the topology of electrical circuits, represented by Tellegen' Theorem, which relies exclusively on Kirchhoff's laws and the topology of the circuit, without regard to the nature of the elements, with Hamiltonian mechanics, which relies on the existence of a coherent structure that links the generalized coordinates to the accompanying moments in phase space, represented by the Hamiltonian.. We demonstrate that Tellegen's theorem is not merely a conservation of energy, but rather the topological constraint that generates the symplectic structure of the system. We derive the net Hamiltonian of the storage elements from the time-integration of Tellegen's theorem and prove that zero in Tellegen's relation is the topological expression of the least-action principle and is equivalent to the time-derivative of the Hamiltonian, $dH/dt = 0$. We demonstrate how the graph incidence matrix includes the symplectic matrix, which leads to phase space constancy, and that the graph is the true architect of the quantum phase space. We show that the symplectic structure is the classical formulation of Heisenberg's quantum uncertainty principle; ensuring the symplectic structure of the system necessarily entails the fulfillment of the uncertainty principle. We attempt to propose a quantization of Tellegen's theorem for use in quantizing electrical circuits used to fabricate qubits in superconducting processors such as Transmon and others.

Keywords: Tellegen's Theorem, Hamiltonian, Symplectic Structure, Quantum Quantization, superconducting qubit.

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اشتقاق الهاملتوني والبنية التماسكية من نظرية تلجن كمدخل للتكميم الكمومي للدارات الكهربائية

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□ ملخّص □

نقدم في هذا البحث إطارا نظرياً جديداً يربط بين طوبولوجيا الدارات الكهربائية التي تمثلها نظرية تلجن التي تعتمد حصرياً على قوانين كيرشوف وطوبولوجيا الدارة، من دون اعتبار لطبيعة العناصر، وميكانيك هاملتون الذي يعتمد على وجود بنية تماسكية التي تربط الاحداثيات المعممة بالعزوم المرافقة في فضاء الطور ممثلة بالهاملتوني. نثبت أن نظرية تلجن ليست مجرد انحفاظ للطاقة، بل هي القيد الطوبولوجي المولد للبنية التماسكية للنظام. نشتق الهاملتوني الصافي لعناصر التخزين من مكاملة نظرية تلجن بالنسبة للزمن، ونثبت أن الصفر في علاقة تلجن هو التعبير الهندسي عن مبدأ الفعل الأقل، ومكافئ لاشتقاق الهاملتوني بالنسبة للزمن dH/dt = 0. ونوضح كيف تتضمن مصفوفة الأثر في مخطط البيان incidence matrix in Graph Theorem المصفوفة التماسكية التي تؤدي إلى ثبات المساحة الطورية، وأن مخطط البيان (الغراف) هو المهندس الحقيقي لفضاء الطور الكمومي، ونبين أن البنية التماسكية هي الصيغة الكلاسيكية لمبدأ عدم اليقين الكمومي لهايزنبرغ، فضمان البنية التماسكية للنظام يفرض حتماً تحقق مبدأ اللايقين. ونحاول أن نقدم مقترحاً لتكميم نظرية تلجن لاستخدامها في تكميم الدارات الكهربائية المستخدمة لتصنيع الكيوبت في المعالجات التي تعتمد مبدأ الناقلية الفائقة مثل الترانسمون وغيره.

الكلمات المفتاحية نظرية تلجن، الهاملتوني، البنية التماسكية، التكميم الكمومي، كيوبت الناقلية الفائقة.

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Introduction

The architecture of most quantum computers today is based on the quantum qubit, which relies on the principle of superconductivity. The superconducting qubit has gained prominence for several reasons, most notably its reliance on conventional electrical circuit components such as capacitors, inductors, and Josephson junctions. It also boasts design flexibility and significant scalability [1]. By reducing the temperature of a conventional, non-dissipating circuit to near absolute zero (around 20 m°K, a temperature dissipation much smaller than the energy level distance $K_B T \ll \hbar\omega$), all its wires become superconducting. Through the precise design of electrical components such as capacitors, inductors, bus lines, and Josephson junctions, researchers have been able to construct circuits with symplectic quantum energy levels, resembling controlled artificial atoms. Their importance stems from several considerations, most notably their pivotal role in the design and development of the qubits necessary for building a superconducting quantum computer [2-6].

Recent years have witnessed remarkable progress in superconducting circuits used in quantum systems, spurring increased interest in analyzing and studying their Lagrange and Hamiltonian formulations [7-8]. Although the standard method [9] has been successful in describing many simple circuits containing both linear and nonlinear elements, it faces significant challenges when dealing with more complex circuits that simultaneously contain nonlinear elements of different types.

Research Problem: The quantization of conventional electrical circuits begins with extracting the Lagrange of the circuit based on its kinetic and potential energy, and then obtaining the Hamiltonian based on the circuit's Lagrange [9]. However, for complex electrical circuits containing nonlinear elements, such as Josephson junctions whose energies depend on the flux [10], and quantum phase-slip junctions [11] whose energies depend on the charge, it becomes impossible to consistently express the Lagrange with only one type of variables. This challenge has led researchers to develop geometric approaches to address these complex cases, such as those based on symplectic geometry [12] and the Faddeev-Jackiw reduction method [13], which provides systematic mechanisms based on differential geometry for constructing the Hamiltonian description. Although these methods for extracting the Hamiltonian of superconducting circuits are accurate and reliable, they suffer from significant mathematical complexity, which may limit their applicability, especially to circuits with complex structures.

Research Objective: In this research, we present a new framework for calculating Hamiltonian equations directly from Tellegen's Theorem, known for its geometric simplicity and its advantage in conserving energy. Tellegen's Theorem relies on the topological structure of the circuit (the way the elements are connected to each other) regardless of the values of the elements in the circuit. This topological structure is directly related to the symplectic property of Hamiltonian mechanics in phase space. From this property, which is common to Tellegen's Theorem, Graph Theorem, and Hamiltonian mechanics, we demonstrate a direct connection to Heisenberg's uncertainty principle.

The research aims to move beyond the traditional interpretation of power, proving that zero in Tellegen's equation guarantees the system's survival within the symmetric manifold.

In this research, we will propose a method for quantizing Tellegen's Theorem for direct application in quantizing electrical circuits, thus facilitating a direct transition from Kirchhoff's to Schrödinger's Theorem.

Research Methodology: From Tellegen's Theorem to Heisenberg's Uncertainty Principle

The research methodology is summarized in the following steps.

1. Prove that the integral constant of Tellegen's Theorem is the circuit's Hamiltonian, and that zero in Tellegen's equation is a product of the time derivative of the Hamiltonian. Therefore, this zero represents the law of conservation of energy.
2. Generalize Hamiltonian Theorem to encompass a complete framework for modeling electrical circuits, regardless of their complexity.
3. Formulate the symplectic structure of Hamiltonian Theorem and derive the symplectic matrix *J* from Graph Theorem, which is represented by Tellegen's Theorem. In other words, prove that Tellegen's equation inherently possesses symplecticity.
4. Use Graph Theorem to eliminate excess degrees of freedom, thereby ensuring the integrity of the circuit's symplectic structure through the *J*-matrix, thus guaranteeing a balanced flow of energy between the system's components.
5. Demonstrate that the *J*-matrix implies phase spacing constancy and satisfies Liouville's Theorem of volume in phase space.
6. After ensuring phase spacing constancy, proceed to Heisenberg's uncertainty principle.
7. Proposing the quantization of Tellegen's Theorem for direct use in superconducting qubit design.

Throughout this methodology, we will demonstrate that Tellegen's Theorem transcends its role as a mere tool for calculating power balance in electrical circuits, revealing a profound topological structure that connects topological network geometry to Hamiltonian dynamics. Through the *A* Incidence Matrix, the graph imposes a geometric constraint that ensures the orthogonality of the potential and current spaces. This orthogonality is the physical expression of the symplectic zero that preserves the Hamiltonian. The zero in Tellegen's relation is, in essence, the time derivative of the total energy of a system is zero when the system is **isolated** from external work, and the forces within the system are **conservative** ($dH/dt = 0$), making Kirchhoff's laws the guardian of the symplectic structure. In sophisticated systems like the transmon circuit, this Theorem emerges as a crucial bridge, transforming classical topological relations into quantum equations of motion, the circuit is redefined not merely as a technical device, but as a physical junction operating according to the principle of conservation of energy.

2- Tellegen's Theorem

Tellegen's theorem is unique in that it is entirely based on Kirchhoff's laws and the topological structure of the electrical network. Therefore, it can be applied to any electrical network that obeys Kirchhoff's laws, regardless of its nature, linear or nonlinear, classical or quantum, commutative or non-commutative, time-varying or constant, and its initial conditions are arbitrary. This theorem states that the sum of the total electrical power transferred to all elements of the electrical circuit is zero. This property has fundamental implications, as it requires that the algebraic sum of the products of the voltages and currents in the circuit be equal to zero, as follows [14-16]:

$$\sum_{k=1}^{n} v_k(t)\, i_k(t) = 0 \qquad (1)$$

Where v_k and i_k represent the voltage and current in the same branch, and n represents the number of branches.

In electrical circuit Theorem, a circuit is represented by a graph G, consisting of nodes and branches. We use the incidence matrix A to describe the connections between electrical components [17]. Kirchhoff's law for currents, KCL, is given by the circuit diagram: Ai =

0. Kirchhoff's law for voltages, KVL, is given by: $v = A^T e$ (where e is the node voltage vector).
Tellegen’s theorem is mathematically derived using the graph by the dot product of the voltage and current vectors as follows:

$$v^T i = (A^T e)^T \ i = e^T \ A \ i \tag{2}$$

Since $Ai = 0$ according to Kirchhoff's law, the result is necessarily zero. This means that zero in Tellegen’s equation is not simply a null number, but rather a result of the perpendicularity of the potential space (generated by the graph) and the current space.

2.1. Derivation of the Hamiltonian from the Integration of Tellegen’s Theorem

By taking the indefinite integral of both sides with respect to time (or taking the integral from t_0 to t) of Tellegen’s equation (Eq. (1)), the following is obtained:

$$\int \left(\sum_{k=1}^{n} v_k(t) i_k(t) \right) d\tau = \int 0 d\tau \tag{3}$$

The following is obtained:

$$\sum_{k=1}^{n} \int v_k i_k d\tau = C \tag{4}$$

Where C is the constant of integration.

Physical meaning: It represents the total energy supplied or absorbed to branch k at time t. Tellegen’s theorem states that the sum of these energies represents the total energy of the system (energy stored in power and primary energy storage elements). Therefore, the constant of integration C represents the total energy of the system.

Example of an *LC* circuit with no loss elements (such as resistors):

If we have an *LC* circuit with no loss elements such as resistors, the energy stored in the circuit is written as the sum of the kinetic and potential energy as follows:

$$W(t) = \frac{1}{2} \sum C v^2 + \frac{1}{2} \sum L i^2 \tag{5}$$

The rate of change of stored energy with respect to time (only for energy storage elements) is produced as follows:

$$\frac{W(t)}{dt} = \sum_{k} v_k i_k \tag{6}$$

The right-hand side represents Tellegen’s theorem, therefore Tellegen’s theorem makes the left-hand side (the rate of change of stored energy) equal to zero. That is, the integration constant is equal to the total energy stored in the circuit, i.e., W(t) = C. Since the total energy in conservative systems is equal to the Hamiltonian (the sum of the kinetic and potential energy), we conclude that the integration constant is the Hamiltonian of the circuit:

$$\mathbf{C = E_{Total} = H} \tag{7}$$

Proof using an LC circuit: Assuming a capacitor voltage v_C and an inductor current i_L, according to Tellegen’s theorem and considering the relationship between the capacitor current and voltage, and the inductor voltage and current, we get :

$$v_L i_L + v_C i_C = 0$$
$$i_C = C_{cap}\frac{dv_C}{dt}, \quad v_L = L\frac{di_L}{dt} \tag{8}$$
$$L i_L \frac{di_L}{dt} + C_{cap} v_C \frac{dv_C}{dt} = 0$$

The integration of the two elements results in the following:

$$\frac{1}{2}L i_L^2 + \frac{1}{2}C_{cap} v_C^2 = C \tag{9}$$

This constant is precisely the Hamiltonian, $H = \dfrac{\phi^2}{2L} + \dfrac{q^2}{2C_{cap}}$, where ϕ is the flux of the inductor, and q is the charge of the capacitor. Therefore, the integral constant C is the Hamiltonian *H*.

In classical mechanics, if the Hamiltonian H is not explicitly dependent on time, $\dfrac{dH}{dt} = 0$, the overall derivative of the Hamiltonian with respect to time is given by the following [19]:

$$\frac{dH}{dt} = \sum\left(\frac{\partial H}{\partial q_k}\dot{q}_k + \frac{\partial H}{\partial p_k}\dot{p}_k\right) = 0, \qquad \frac{dH}{dt} = 0, \quad \Rightarrow \quad H = Const = E \tag{10}$$

In electrical circuits, charges q_k represent the generalized coordinates and their currents $\dot{q}_k = i_k$, while fluxes q_k represent the generalized momentum and their potentials $\dot{\phi}_k = v_k$. Comparing Eq. (1) with the energy conservation equation $dH/dt = 0$, we find that Tellegen’s equation is precisely the physical representation of the time derivative of the Hamiltonian in circuit space. The zero in Tellegen’s equation ensures that the system is energy-closed, meaning that the power generated by the sources is exactly equal to the power stored or consumed by the elements. $H = E$ represents the surface in phase space, and the flux and charge move along this surface.

Physical conclusion: The integration constant is exactly equivalent to the total energy in the system and exactly equivalent to the Hamiltonian in energy-conservative systems.

- The zero in Tellegen’s equation means that the currents and potentials exist in topologically orthogonal spaces. This orthogonality ensures that the virtual work done by the internal forces is zero. In mechanics, virtual work is fictitious energy that we experience at a very small displacement without changing equilibrium. Here, currents are treated as "displacements" and voltages as forces. Since the currents and voltages are in perpendicular spaces, any virtual change in current does not produce work from the internal voltages, and vice versa. This zero is what ensures that the circuit is in equilibrium, and that the responses (currents and voltages) are consistent with each other without producing fictitious energy out of nothing.
- Since zero signifies perpendicularity between potential and current vectors, this zero represents the perpendicularity between charge and flux.
- Zero is derived from the time derivative of the Hamiltonian, thus representing the law of conservation of energy.
- This zero embodies the principle of least action (Hamilton's principle) disguised in geometric terms.
- This zero guarantees that generalized coordinates do not interfere with generalized moments in a way that violates the laws of physics.

- This zero will have additional meanings when quantizing Tellegen's Theorem, which we will demonstrate in our next discussion.

From this summary, we conclude the validity and effectiveness of Tellegen's Theorem in quantizing electrical circuits.

2-2-Example: Hamiltonian Derivation of a Transmon Qubit:

The Transmon circuit is a Qubit, where is developed for the CPB (Cooper Pair Box) circuit (Figure 1), consisting primarily of a Josephson junction connected in parallel with a large capacitor (CB) for noise reduction. The Josephson junction acts as a non-linear inductor. This type of non-linearity is necessary to make the power levels unequal in dimensions, allowing us to control only the first two levels to represent the qubit [20].

Derivation of the Hamiltonian via Tellegen's theorem: Applying Tellegen's theorem to the capacitor branch and the junction branch:

$$I_{cap}V + I_J V = 0$$

We know that the current and voltage of a Josephson junction are given by the following [20]:

$$I_J = I_c \sin(\varphi), \quad V = \frac{\phi_0}{2\pi}\dot{\varphi}$$

Where φ is the difference in phase between the wave functions in the two superconductors. In a Josephson junction, we have $\phi_0 = \hbar / 2e$ the magnetic flux quantum (the smallest unit of magnetic flux in the junction), I_c the critical current coefficient of the junction $\phi = 2\pi\, \phi / \phi_0$, and the quantized phase difference across the junction. By substitution in the Tellegen's relation, the following is obtained:

$$C_J \frac{dV}{dt} V + I_c \sin(\varphi) \frac{\phi_0}{2\pi}\dot{\varphi} = 0$$

Hamiltonian produces the transmonian qubit as follows:

$$H = 4E_C (n - n_g)^2 - E_J \cos(\varphi)$$

Where E_C is the charge energy, E_J is the Josephson energy, n is the Cooper pair number operator, and n_g represents the charge displacement when a DC voltage is applied

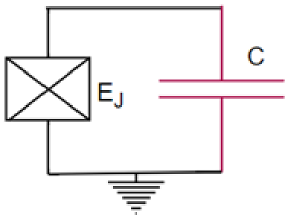

Figure 1: The Transmon Qubit circuit

The nonlinearity of the Josephson junction does not break the Tellegen's relation, but rather evolves the phase space in a way that allows the qubit to appear while the Tellgen's zero remains constant and unchanging.

2-3- Generalization of the Hamiltonian

The derivation of the Hamiltonian from Tellegen's theorem can be generalized to encompass the entire circuit, allowing us to obtain the Hamiltonian for any circuit, regardless of its complexity.

The overall Hamiltonian can be written as follows:

$$H_{Total} = H_{sys} + H_{\text{int}} + H_{env} \tag{11}$$

Where H_{sys} is the Hamiltonian of the basic elements in the circuit, H_{int} is the interaction Hamiltonian of the magnetic elements, and H_{env} is the Hamiltonian resulting from the interaction with the surrounding medium (environment). The relationship of the overall Hamiltonian can be detailed as follows:

$$H_{Total} = \underbrace{H_{LC} + H_J + H_P}_{H_{nodes}} + \underbrace{H_{src}}_{H_{ext}} + \underbrace{H_{line} + H_{mut}}_{H_{field}} + \underbrace{H_{diss}}_{H_{bath}} \tag{12}$$

Where:

H_{LC} (Hamiltonian of Energy Elements): This represents the energy stored in linear elements (capacitors and inductors). It is the cornerstone, directly derived from the Tellegen's integral of an *LC* circuit.

H_J (Hamiltonian of Josephson Junction) and H_P (Hamiltonian of Phase Slip): This is one of the most important limits in quantum circuits, as it adds the nonlinear component (sinusoidal potential) that allows for the quantization of energy levels.

H_{mut} (Hamiltonian of Magnetic Exchange): This represents the energy stored in the mutual induction between branches and appears in Tellegen's theorem as a coupling term between different branch currents.

H_{line} (Hamiltonian of Transmission Lines): This is usually treated as a continuous Hamiltonian (or a set of harmonic oscillators) if the lines are long relative to the wavelength. It is essential in microcircuits.

H_{src} (Hamiltonian of Sources): This represents the interaction with the external environment and is often a time-dependent limit ($V_s(t),q$). The fundamental problem in representing the Hamiltonian resulting from the dissipating elements (resistance) remains. As is known, resistance transforms the Hamiltonian system from a closed, energy-conserving system to an open one, where resistance depletes the energy stored in the storage elements and dissipates it as heat.

Several models are used to represent resistance in a Hamiltonian system; we will choose the Calderia-Legget path model [4]. In this model, we do not write $H = R\,.i^2$, because this expression represents the dissipated power, not the stored energy. Instead, resistance (dissipation) is represented as a set of small harmonic oscillators that absorb energy from the system and do not return it. In this sense, H_{diss} becomes the Hamiltonian of this environment/bath. Representing Transmission Lines in Tellegen's Theorem

Instead of treating transmission lines as abstract entities, a transmission line can be divided into a series of infinitesimal sections, each represented by an inductor $L\Delta z$ and a capacitor $C\Delta z$.

When Tellegen's theorem is applied to this series, the discrete sum becomes an integral over the line z, as follows:

$$\int_0^d \left(v(z,t)\frac{\partial i(z,t)}{\partial z} + i(z,t)\frac{\partial v(z,t)}{\partial z} \right) dz = 0 \tag{13}$$

This integration leads directly to the classic Hamiltonian transmission line as follows:

$$H_{line} = \int_0^d \frac{1}{2}\left(L_0 i^2(z,t) + C_0 v^2(z,t) \right) dz \tag{14}$$

Where L_0 and C_0 are the inductance and capacitance per unit length.

In conclusion, we have constructed the general Hamiltonian, which appears as a modular hierarchical structure as follows:

- The Core: Represented by the Hamiltonian $H_{node} = H_{LC} + H_J + H_p$, it includes the central components such as capacitors, inductors, Josephson junctions, and phase-slip junctions.
- The Link: Represented by $H_{mut} + H_{line}$, it includes the magnetic interactions and the transmission lines that connect the components.
- The Environment: Represented by $H_{ext} + H_{diss}$, it includes the sources and dissipators. Tellegen's Theorem ensures a consistent flow of energy between the system and its environment.

Table (1) provides a general summary of the circuit components, the nature of the Hamiltonian, the role of the Tellegen's equation, and the equivalent mechanical description.

Tellegen's Theorem is the ideal tool for circuit partitioning. While the energy-conserving part of the Tellegen (the energy-storing elements for producing the Hamiltonian) deals with the energy input and output fluxes of this Hamiltonian, the other part (resistors and sources) deals with the input and output fluxes of this Hamiltonian. This allows Tellegen's Theorem to differentiate between the system's structure and external influences. From Table (1), we can summarize the following facts:

Table (1) A general summary of the circuit elements, the nature of the Hamiltonian, the role in the Tellegen's equation, and the equivalent mechanical description.

Mechanical Description (Analogy)	Role in the Tellegen's equation $\sum vi = 0$	Hamiltonian Nature	Circuit Element
Potential Energy (Spring)	Storage Branch $vC \cdot \frac{dq}{dt}$	Electric Potential Energy $\frac{q^2}{2C}$	Capacitor (*C*)
Kintic Energy (Mass)	Storage Branch $\phi \cdot \frac{d\phi}{dt}$	Magnetic Kinetic Energy $\frac{\phi^2}{2L}$	Inductor (*L*)
Nonlinear Pendulum	Nonlinear (Josephson Tunnel Current)	Nonlinear Potential Energy $-E_J \cos\varphi$	Josephson Junction
Dambling and Friction	$R.i^2$ (Power Dispersion)	Dissipation Function (Not Stored Energy)	Resistance (*R*)
Optimal Strings and Waves	Continuous (Differential) Branches	Distributed Field Energy (Integral C_0, L_0)	Transmission Lines
External Forces	Branches of power injection or withdrawal from the system External Forces	External Interaction Energy ($-qVs$) and ($-\phi Is$)	Sources (*Vs, Is*)

Tellegen's Zero as a Balance: We find that the Tellegen's zero acts as a balance, where the power in the storage branches (the time derivative of the Hamiltonian) is equal to the power in the dissipation and source branches.

Topological Structure: Note that the third column (Tellegen) depends only on how these elements are connected (the graph), while the second column (the Hamiltonian) depends on the physical properties of each element.

Circuit Quantization: When transitioning to transmon, this table remains valid, with the classical variables (q, ϕ) being replaced by quantum operators $(\hat{n}, \hat{\varphi})$ [20].

3- Graph Theorem and Degrees of Freedom

The Hamiltonian represents the heart of any physical system, completely determining the system's behavior. However, when we deal with complex or interconnected systems, writing a Hamiltonian directly becomes a cumbersome process, as it involves a vast number of degrees of freedom, many of which are constrained or redundant and do not contribute to the intrinsic physical picture.

To address this problem, we present our method for removing the redundant degrees of freedom to obtain a generalized Hamiltonian formulation, and we illustrate this method using the circuit shown in Figure (2-a).

(1): We convert the electrical circuit into a directed graph, as shown in Figure (2-b), where the nodes represent the connection points and the branches represent the physical elements. This graph forms the topological basis that defines all possible connections.
(2): We then choose a normal tree, which is a sub-graph that is interconnected and contains all the nodes in the graph but has an infinite path. In our example, the normal tree is constituted from the branches: T = {1, 2, 4, 6}. Next, we find the branches of the auxiliary tree (co-tree), which contains the remaining branches not found in the normal tree. In our example, the auxiliary tree is co-tree = {3, 5}.

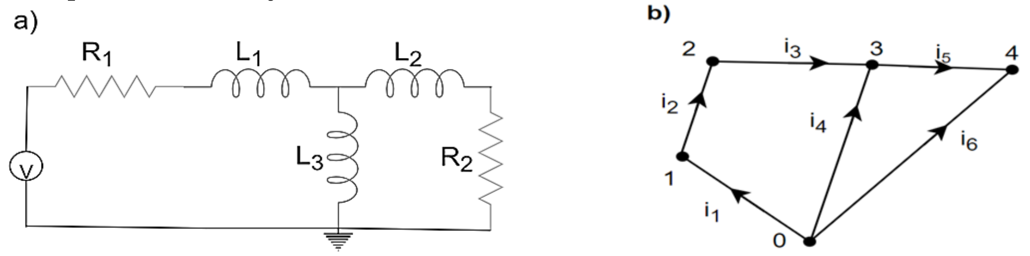

Figure (2): (a) An optional electrical circuit, and (b) The graph of this circuit.

We take one branch from the auxiliary tree and the remaining branches from the normal tree to form loops. In our example, we obtain two loops:

loop_1 = {1, 2, 3, 4}, loop_2 = {4, 5, 6}

(3): We write the equations for the two loops, taking into account the counterclockwise direction, using Kirchhoff's second law (KVL), which expresses the potential relationships between the branches as follows:

$$\sum v = 0 \tag{15}$$

The equation for the first loop is:

$$-V - v_2 - v_3 + v_4 = 0, \qquad -V - I_2 R_1 - L_1 \frac{dI_3}{dt} + L_3 \frac{dI_4}{dt} = 0 \tag{16}$$

The equation for the second loop is:

$$-L_2 \frac{dI_5}{dt} + I_6 R_2 - L_3 \frac{dI_4}{dt} = 0 \tag{17}$$

(4): We write the current equations using Kirchhoff's node law (KCL), which expresses the linear relationships that connect branch currents to nodes, as shown in the relationship:

$$\sum I = 0 \tag{18}$$

$$I_2 - I_3 = 0, \qquad I_3 + I_4 - I_5 = 0, \qquad I_5 + I_6 = 0$$

We substitute the equations of the nodes into the equations of the loops, so the equation of the first loop is as follows:

$$-L_1 \frac{dI_3}{dt} + L_3 \frac{dI_4}{dt} = V + I_3 R_1 \tag{19}$$

The equation of the second loop is as follows:

$$-L_1 \frac{dI_3}{dt} - (L_1 + L_1) \frac{dI_4}{dt} = I_3 R_2 + I_1 R_2 \tag{20}$$

The last two equations show that the degree of freedom is 2 and that inductor L_1 and L_2 are not linearly independent. Therefore, the proposed equivalent circuit is as shown in Figure (3).

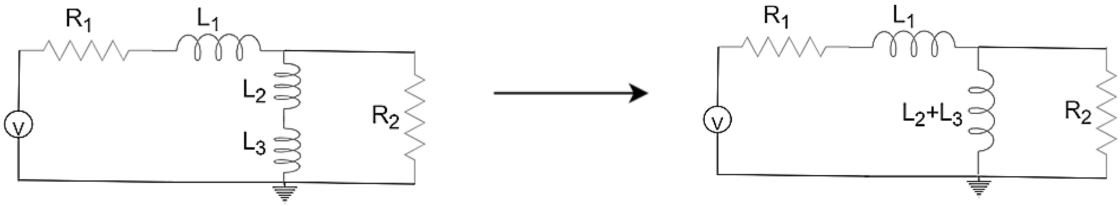

Figure (3): The equivalent circuit after redundant freedom removed

4-The Symplectic Formulation of the Hamiltonian

Hamilton's equations can be expressed in matrix form using the symplectic form [19]. If we have a *2n*-element normal vector, z_1, z_2, ..., z_{2n}, it is defined as follows:

$$z_i = q_i, \qquad z_{n+i} = p_i, \qquad i = 1,2,\ldots,n \tag{21}$$

The first *n* elements of the vector *z* represent n trace elements of the *q* generalized coordinate, and the last *n* elements of the vector *z* represent the generalized momentum *p*. The normal vector of the derivative of the Hamiltonian with respect to the vector *z* is given by [19]:

$$\left(\frac{\partial H}{\partial z}\right)_i = \frac{\partial H}{\partial q_i}, \qquad \left(\frac{\partial H}{\partial z}\right)_{n+i} = -\frac{\partial H}{\partial p_i}, \qquad i = 1,2,\ldots,n \tag{22}$$

Let us define the *2n*x*2n* matrix *J* as follows:

$$J = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -I_{n\times n} & 0_{n\times n} \end{bmatrix} \tag{23}$$

Where *J* is a skew-symmetric matrix, *0* is a zero matrix of dimensions *n*x*n*, and *I* is an identity matrix of dimensions *n*x*n*. The matrix J is called the fundamental symplectic matrix.

Hamilton's equations can be expressed in symplectic form and in matrix form as follows:

$$\dot{z} = J\,\frac{\partial H}{\partial z} \tag{24}$$

The matrix *J* is used to test whether a matrix has a symplectic structure. The matrix Ω is a symplectic matrix if the following condition is met:

$$J^T \Omega\, J = \Omega \tag{25}$$

For the LC oscillatory circuit, we found that the Hamiltonian function is given by:

$$H = \frac{1}{2}\frac{\phi^2}{L} + \frac{1}{2}\frac{q^2}{C} \tag{26}$$

Therefore, Hamilton's equations are written in matrix symplectic form as follows:

$$\begin{bmatrix} \dot{q} \\ \dot{\phi} \end{bmatrix} = J \begin{bmatrix} \dfrac{\partial H}{\partial q} \\ \dfrac{\partial H}{\partial \phi} \end{bmatrix} \;\Rightarrow\; \begin{bmatrix} \dot{q} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{q}{C} \\ \dfrac{\phi}{L} \end{bmatrix} \tag{27}$$

The physical meaning of the symplectic property: If the system has a symplectic structure, meaning the matrix *J* is skew-symmetric ($J = -J^T$), this means that the phase space is constant (Liouville's theorem of volume is satisfied). That is, when performing quantization, we must ensure this constancy of the phase space so that Heisenberg's uncertainty principle holds true when quantizing Tellegen's Theorem. This research will show that Tellegen's Theorem, as a Theorem based on the topology of the system, includes the symplectic property in its structure [19].

5- Derivation of the Symplectic Structure from Graph Theorem

To demonstrate that the topology represented by Tellegen's Theorem is the source of the symplectic structure, we will use Hamilton's symplectic formulation in terms of energy variables in phase space.

Representation of Energy Variables in Phase Space: We define the vector of the variables using the symplectic variables (fluxes ϕ and charges q), as follows:

$$z = \begin{bmatrix} q \\ \phi \end{bmatrix} \tag{28}$$

Where q represents the generalized coordinates, and ϕ represents the generalized moments.

The Incidence Matrix in Graph Theory is a mathematical table that describes the relationship between nodes and branches (resistors, inductors, capacitors, and voltage sources) in an electrical circuit. Its explanation in a few lines:

Construction: Each row in the matrix represents a node, and each column represents a branch (an electrical component). Values: Each component is assigned a sign as follows:

- (+1) If current flows from the branch to the node.
- (-1) If current flows into the node.
- If the branch is not connected to that node.

Using the Incidence Matrix A in Kirchhoff's Laws: From Graph Theorem, branch variables are related to potentials and currents via the Incidence matrix A (or topological connection matrix B, the loop matrix)

Current Law KCL: $Ai = 0 \Rightarrow \quad i \in null(A)$

Potential Law KVL: $v = A^T e \quad \Rightarrow \quad v \in rang(A^T)$

Where A^T is the transposed matrix of matrix A.

Since $I = \dot{q}$ and $v = \dot{\Phi}$, Graph Theorem assumes that the derivatives of symplectic variables exist in topologically orthogonal spaces, which is confirmed by Tellegen's theorem:

$$\dot{\phi}^T \cdot \dot{q} = 0 \tag{29}$$

5-1-Constructing the Symplectic Matrix J

Let's take a simple LC circuit as an example. The incidence matrix assumes that the inductor voltage equals the capacitor voltage (considering the direction), resulting in:

$$\dot{\phi} = -v_C = -\frac{\partial H}{\partial q}, \qquad \dot{q} = i_L = \frac{\partial H}{\partial \phi} \tag{30}$$

Writing this relationship as a matrix, the symplectic matrix J appears as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{q} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}_{J} \begin{bmatrix} \dfrac{\partial H}{\partial q} \\ \dfrac{\partial H}{\partial \phi} \end{bmatrix} \tag{31}$$

5-2-Generalization via the Incidence Matrix

In complex circuits, the symplectic matrix J is derived from the incidence matrix A via the relationship between the tree branches and the links (co-tree branches). If we have a topological link matrix Γ derived from A, which describes how the inductors are connected to the capacitors via the circuit loops, then the overall symplectic matrix takes the following form [18]:

$$J = \begin{bmatrix} 0 & \Gamma \\ -\Gamma^T & 0 \end{bmatrix} \tag{32}$$

Where Γ is the sub-block matrix relating the capacitors and inductors in the circuit (between the branch currents and link currents, and between the link potentials and branch potentials). If the capacitor and inductor are in the same fundamental loop, the values 1 and -1 appear; if there is no correlation, the value (0) appears.
Deriving matrix J from incidence matrix A proves that matrix J is not an external entity imposed on the circuit. Rather, it is contained within the circuit topology represented by Tellegen's theorem. Incidence matrix A reflects how energy is distributed between electrical energy storage elements (capacitors) and magnetic energy storage elements (inductors). Since the link matrix Γ between energy storage elements is derived from the Graph, the phase-area stability provided by J is actually a topological stability imposed by Tellegen's theorem.

5-3-Generalization of the topological generator of the symplectic structure from incidence matrix *A* to matrix *J*

In Graph Theorem applied to circuits, we not only use incidence matrix A as a pure matrix, but also decompose it based on the circuit's spanning tree. This decomposition determines how the capacitors and inductors exchange energy, and thus determines the structure of matrix J.

Graph Tree-Link Decomposition: We choose a spanning tree so that we place as many capacitors as possible in the tree branches and inductors in the link/co-tree links, similar to the excess degrees of freedom removal algorithm explained in the previous section. Based on this division, the incidence matrix A can be reduced to the fundamental loop matrix, denoted by B. This matrix represents how the branches are topologically connected:

$$B = \begin{bmatrix} I & \Gamma \end{bmatrix} \tag{33}$$

Where Γ represents the topological link matrix, which describes the interdependence between the potential elements (tree) and the current elements (links), and I represents the unitary matrix.

Relation to Tellegen's Theorem: The Tellegen's relation in its matrix form, $V^T i = 0$, can be rewritten using the link matrix Γ as follows:

$$v_{links} = -\Gamma^T v_{tree}, \tag{34}$$
$$i_{tree} = -\Gamma i_{links}$$

These equations show that the graph acts as a topological transformer, transferring potentials from the tree to the links and currents from the links to the tree.

Constructing the Symplectic Matrix *J*: Since we have defined the symplectic variables such that the charges q are associated with the tree capacitors, and the fluxes ϕ are associated with the link inductor, the derivatives of these variables (via Kirchhoff's laws derived from Γ) become as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{q} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \Gamma \\ -\Gamma^T & 0 \end{bmatrix}}_{J} \begin{bmatrix} \dfrac{\partial H}{\partial q} \\ \dfrac{\partial H}{\partial \phi} \end{bmatrix} \tag{35}$$

Here, the general symplectic matrix J appears as follows:

$$J = \begin{bmatrix} 0 & \Gamma \\ -\Gamma^T & 0 \end{bmatrix} \tag{36}$$

Physical Implication: This matrix J possesses the required properties in Hamiltonian mechanics:

1- Skew-symmetry $J^T = -J$: which ensures the conservation of energy ($H' = 0$) via Tellegen.
2- Full Rank: which ensures that no dimension of the phase space is lost, which is the fundamental condition for the operation of Heisenberg's uncertainty principle.

With this result, we ensure the smooth transition from the graph through the incidence matrix (A) to the link matrix (Γ) to the symplectic matrix (J) to the uncertainty principle.

Example of a Third-Order Circuit: Circuit Description: The circuit consists of: A first capacitor (C_1) in the first branch**,** an inductor (L_2) connecting the two capacitors (link), and a capacitor (C_3) in the second branch.

Topological Link Matrix Γ: In this circuit, the inductor L_2 forms a primary link with capacitors C_1 and C_3. Applying Kirchhoff's laws of currents and voltages, we find that the link matrix Γ, which relates the link current (inductor) to the branch currents (capacitors), yields the following:

$$\Gamma = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{37}$$

This means that the inductor current flows in one direction through the first capacitor and in the opposite direction through the second.

Constructing the Symplectic Matrix *J* for a Third-Order Circuit The state vector here consists of three variables: two charges and one flux, $z = [q_1, q_3, \phi_2]^T$. Applying the general relationship of the symplectic matrix (Eq. (36)), we obtain the following:

$$J = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \tag{38}$$

Hamilton Equations: The Hamiltonian for this circuit, according to Tellegen's theorem, yields the following:

$$H = \frac{q_1^2}{2C_1} + \frac{q_3^2}{2C_3} + \frac{\phi_2^2}{2L_2} \tag{39}$$

Multiplying matrix J by the Hamiltonian gradient $\nabla H = \left[\frac{q_1}{C_1} + \frac{q_3}{C_3} + \frac{\phi}{L_2}\right]^T$, we obtain:

Eq. 1: $\dot{q}_1 = \frac{\phi_2}{L_2}$ (The current of the first capacitor is the current of the inductor.

Eq. 2: $\dot{q}_3 = -\frac{\phi_2}{L_2}$ (the second capacitor current is the opposite of the current of the inductor.

E. 3: $\dot{\phi}_2 = -\frac{q_1}{C_1} + \frac{q_3}{C_3}$ (The potential difference between the two capacitors).

This example demonstrates that the skew-symmetric matrix J is responsible for the energy exchange between the elements. The numbers (1, -1) within matrix J do not originate from physical values (L, C), but rather from the geometry of the connection (graph).

6-Symplectic Structure and the Uncertainty Principle

This research has proved that the integral of Tellegen's Theorem is the Hamiltonian of the electrical circuit, and we showed that zero in Tellegen's relation is a result of the time-dependent Hamiltonian in energy-conservative systems. We then verified that the Graph Theorem, represented by Tellegen's Theorem, possesses the symplectic property (the symplectic matrix J), which leads to the constancy of the phase space. This is exactly equivalent to the realization of Liouville's theorem regarding the constancy of volume in Hamiltonian systems in phase space.

In this section, we explain how to move from the symplectic structure to Heisenberg's uncertainty principle according to the following steps:

1-Tellegen's Theorem: Through Tellegen's Theorem, we proved that the variables (voltages and currents) are constrained by the Graph topology. Since the incidence matrix *A* imposes orthogonality between the intensive and large energy variables, it generates the symplectic transformation matrix *J*, which relates the generalized coordinates *q* to the generalized momentum *p*. This matrix is the skeleton of the phase space. Mathematically, Tellegen's theorem states that the potential vector *v* and the current vector *i* are always orthogonal due to the incidence matrix *A*:

$$v^T \cdot i = 0, \quad \Rightarrow \quad \dot{\phi}^T \cdot \dot{q} = 0 \tag{40}$$

Since $I = \dot{q}$ and $v = \dot{\phi}$, the graph assumes that the derivatives of symplectic variables exist in topologically orthogonal spaces, which is what Tellegen's theorem achieves: Tellegen's theorem imposes a constraint on the derivatives of variables in the phase space.

2-Graph Generates Symplectic Structure: In conservative Hamiltonian systems, the Tellegen's relation can be expressed by the transformation matrix between the Hamiltonian gradient and the derivatives of the variables as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H}{\partial q} \\ \dfrac{\partial H}{\partial \phi} \end{bmatrix}, \quad where \quad J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{41}$$

This matrix *J* is a skew-symmetric matrix, and its generation depends entirely on how the branches are connected (topology) in a way that ensures the law of conservation of energy:

$$\frac{dH}{dt} = 0 \tag{42}$$

3-Conservation of Phase Space (Liouville's Theorem): The existence of symplectic structure *J* means mathematically that the system's transformations are symmetric transformations. These transformations are characterized by their preservation of the symplectic 2-form, which is physically equivalent to Liouville's law. The phase space (or volume in phase space) remains constant during the system's time evolution [19].

4-From Phase Space Constancy to the Uncertainty Principle: In classical mechanics, the phase space $dq \wedge dp$ is constant, but it can be very small (approaching zero). When the system is quantized, this space also becomes quantized, since no quantum state can occupy a phase space smaller than Planck's constant *h*. According to Liouville's Theorem, the time-shift governed by the *J*-matrix preserves the two-dimensional symplectic structure ω, as follows:

$$\omega = \sum d\phi_k \wedge dq_k \tag{43}$$

This means that the determinant of the transformation matrix *M* is one, det(*M*) = 1, which ensures that the phase space remains constant and does not contract.

5-The Quantum Result (Uncertainty Principle): Since the Tellegen's Theorem of the *J*-matrix ensures that the phase space does not contract or vanish during the system's motion, it physically guarantees that the phase cell occupied by the electron or Cooper pair will retain its minimum boundary. Therefore, Tellegen's Theorem is the topological guarantor of the phase space's non-collapse during quantization, thus fulfilling Heisenberg's uncertainty principle:

$$\Delta q \, \Delta p \geq \frac{\hbar}{2} \tag{44}$$

Here, the uncertainty principle emerges as a topological necessity to protect the space imposed by Tellegen's Theorem from the outset. Thus, we demonstrate that the graph matrix is not merely a computational tool, but rather the geometric guarantor of information non-loss in the phase space. Without the symplectic structure derived from Tellegen, the uncertainty principle would lack a stable space within which to operate.

6-Quantization of Tellegen's Theorem: Tellegen's Theorem can be quantized by transforming classical variables (voltage and current) into quantum operators in Hilbert space.

In classical circuits, we deal with instantaneous values $v_k(t)$ and $i_k(t)$. In the quantum version, we replace them with voltage $\hat{V}_k$ and current operators $\hat{I}_k$, and Tellegen's theorem is formulated as follows:

$$\sum_{k=1}^{n} \hat{V}_k \hat{I}_k = 0 \tag{45}$$

However, in quantum mechanics, operators are not necessarily interchangeable $[\hat{V}_k, \hat{I}_k] \neq 0$ (order is important for operators). Therefore, when quantizing Tellegen, the order of operators must be considered to ensure that the result (power) is a Hermitian operator representing a measurable physical value. Consequently, Tellegen's theorem is written in symmetric form:

$$\frac{1}{2} \sum_{k=1}^{n} (\hat{V}_k \hat{I}_k + \hat{I}_k \hat{V}_k) = 0 \tag{46}$$

As a principle of conservation of quantum power, Tellegen's theorem provides us with the following new insights:

1- Heisenberg's power relation: Quantization implies that there are limits to the accuracy of measuring voltage and current together in a single branch. However, Tellegen's theorem states that the sum total of these operators across the circuit must remain zero, indicating the existence of quantum correlations between the branches of the circuit imposed by the circuit topology.

2-Topological Entanglement: The quantum zero in Tellegen can be considered to impose a state of entanglement between qubits or quantum elements in the circuit to ensure the conservation of total energy. Since the voltage in superconducting circuits is related to the phase change rate $(\hat{V} \propto \frac{d\hat{\varphi}}{dt})$, Tellegen's quantization imposes a topological constraint on quantum phases. The quantized Tellegen's zero can be considered the guarantee of phase consistency, as it forces the vibrational modes in the circuit to evolve harmonicly over time to prevent any disruption to the overall power balance.

Results and Discussion

Tellegen's Theorem, with its comprehensive coverage of active and passive, linear and nonlinear branches, classical and quantum circuits allows us to construct a complete Hamiltonian for the circuit. Within this framework, the circuit is no longer merely a series of wire connections, but rather a complete physical system, where the Hamiltonian boundaries express the energy distribution between nodes (Josephson junctions and

capacitors), fields (magnetic exchange and transmission lines), and interactions with the surrounding environment (dissipators and sources). Tellegen's Theorem enables us to quantize a simple *LC* circuit into a general framework suitable for modeling any complex electrical system, from the simplest circuits to the most sophisticated quantum processors.
The derivation of the symplectic matrix from Graph Theorem demonstrates that the *J*-matrix is not an external entity imposed on the circuit, but rather a topological signature reflecting the incidence matrix distribution of the connections between electrical energy storage elements (capacitors) and magnetic elements (inductors). Since the binding matrix of the storage elements is derived from the graph, the phase space constancy (encompassed by *J*) is, in fact, a topological constancy imposed by Tellegen's Theorem.
The connection between the topology of the graph and the uncertainty principle is not merely a mathematical coincidence. The symplectic structure imposed by Tellegen's Theorem acts as a shield, preventing the phase space from collapsing. By maintaining a constant phase volume (symmetric invariance), it sets the necessary topological stage for the emergence of quantum laws, where the uncertainty principle becomes an inevitable consequence of the rigidity of the geometric structure designed by the graph
.

Conclusion and Recommendations

In this research, we discussed Tellegen's Theorem as a natural bridge between electrical and analytical engineering by demonstrating that the integral constant of this Theorem is the Hamiltonian of the circuit. We proved that the graph is the common backbone between electrical and Hamiltonian systems. From the properties of the graph, which includes a symplectic matrix that ensures phase space constancy, we found that the transition from Kirchhoff's laws to Heisenberg's uncertainty principle to Schrödinger's equation is direct and seamless. This transition illustrates how the law of conservation of power is transformed from a quantum physical concept to a phase-based informational concept.
The quantization of Tellegen's Theorem opens the door to a deeper understanding of quantum circuits, as the Theorem not only assumes the conservation of energy but also acts as a topological constraint linking the non-exchangeable influences of the elements. This opens avenues for understanding circuits as topological junctions that maintain their symplectic structure. This quantization proves that the geometric structure of the circuit (the graph) remains the ultimate governing factor in dynamics, whether the particles obey Newton's laws or Schrödinger's laws.
Through this research, we have demonstrated the possibility of linking the simplest laws of electricity (Kirchhoff's laws and Tellegen's Theorem) with the most profound law of quantum physics: the uncertainty principle. Following these demonstrations of the connection between electrical engineering and quantum physics, our next research will be a detailed study of the quantization of Tellegen's Theorem, to be applied to the design of superconducting qubits.

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