

Fitting Arabic Letter "Baa" Using Bézier-Like Curves with Two Shape-Parameters Basis

Dr. Nouri Suleiman* 

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□ ABSTRACT □

Arabic font modeling is considered one of the most challenging problems in computational geometry due to the cursive and highly flexible structure of Arabic characters. In this paper, a new trigonometric–exponential basis with two shape parameters is proposed for constructing Bézier-like curves and applying them to Arabic font fitting. The proposed basis preserves the fundamental geometric properties of classical Bernstein basis functions, including non-negativity, partition of unity, endpoint interpolation, and convex hull property, while introducing additional flexibility through adjustable shape parameters. The influence of the parameters on the geometric behavior of the curve is analyzed. A least-squares fitting method is employed to approximate contour data extracted from the Arabic letter “Baa”.

Numerical experiments demonstrate that the proposed Bézier-like curve gives more accurate fitting results than the classical cubic Bézier curve in several cases, particularly when suitable values of the shape parameters are selected. The proposed approach also improves local shape control without increasing the degree of the curve or the number of control points. The results indicate that the introduced basis is effective for digital typography, Arabic calligraphy reconstruction, geometric modeling, and computer-aided font design.

Keywords: Bézier-like curves, trigonometric basis, least-squares method, curve fitting.

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* Associate Professor, Department of Mathematics, Faculty of Science, Idlib University, Idlib, Syria.

مواصفة الحرف العربي "باء" باستخدام منحنيات شبيهة بيزييه ذات قاعدة بمعلمتي شكل

د. نوري سليمان*

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□ ملخص □

تُعدّ نمذجة الخط العربي من أكثر المشكلات تحديًا في الهندسة الحسابية، وذلك بسبب البنية المتصلة والمرنة للغاية للحروف العربية.

اقترحنا في هذا البحث قاعدة مثلثية أُسيّة جديدة بمعلمتي شكل لبناء منحنيات شبيهة بيزييه وتطبيقها على مواصفة الخط العربي.

تحافظ القاعدة المقترحة على الخصائص الهندسية الأساسية لدوال قاعدة برنشتاين الكلاسيكية، بما في ذلك عدم السلبية، وتقسيم الوحدة، والاستيفاء عند النهايات، وخاصية الغلاف المحدب، مع تقديم مرونة إضافية من خلال معلمتي الشكل القابلتين للتعديل.

تم تحليل تأثير هاتين المعلمتين على السلوك الهندسي للمنحنى، كما استُخدمت طريقة المربعات الصغرى لمواصفة بيانات الحدود الخارجية للحرف العربي "الباء".

أظهرت التجارب العددية أن منحنيات شبيهة بيزييه المقترحة تعطي نتائج مواصفة أكثر دقة مقارنة بمنحنى بيزييه التكعيبي الكلاسيكي في عدة حالات، خصوصًا عند اختيار قيم مناسبة لمعلمتي الشكل، وكذلك تسهم الطريقة المقترحة في تحسين التحكم المحلي بشكل المنحنى دون زيادة درجة المنحنى أو عدد نقاط التحكم. تشير النتائج إلى أن القاعدة المقترحة فعّالة في الطباعة الرقمية، وإعادة بناء الخط العربي، والنمذجة الهندسية، وتصميم الخطوط بمساعدة الحاسب.

الكلمات المفتاحية: منحنيات شبيهة بيزييه، قاعدة مثلثية، طريقة المربعات الصغرى، مواصفة المنحنيات.

حقوق النشر  : مجلة جامعة اللاذقية (تشرين سابقاً) - سورية، يحتفظ المؤلفون بحقوق النشر

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*أستاذ مساعد، قسم الرياضيات، كلية العلوم، جامعة إدلب، إدلب، سورية.

1. Introduction:

Arabic typography represents one of the most complex and aesthetically rich writing systems, characterized by its cursive nature, contextual letter forms, and strong dependence on smooth geometric transitions. Unlike Latin fonts, Arabic characters are inherently connected, and their shapes vary depending on their position within a word, i.e. initial, medial, final, or isolated. This structural complexity makes the digital representation and geometric modeling of Arabic fonts a challenging problem in computational geometry and computer-aided design.

Traditional approaches to font representation often rely on classical polynomial Bézier curves, particularly those based on the Bernstein basis. These methods are widely used due to their simplicity, numerical stability, and intuitive geometric interpretation.

However, recent studies show that classical Bézier formulations may lack sufficient flexibility when modeling highly intricate shapes such as Arabic calligraphy, especially in preserving fine stylistic features and curvature continuity [1,2].

In this research, we propose the use of a trigonometric–exponential Bézier basis for fitting Arabic font shapes. This basis incorporates trigonometric functions combined with exponential weighting parameters, allowing for enhanced control over curve behavior and shape modulation.

Recent advances in geometric modeling emphasize the importance of introducing shape parameters into Bézier-like bases to improve flexibility and local control over curves [2,3].

These developments are particularly suitable for font design, where subtle geometric variations play a crucial role.

The proposed method constructs parametric curves by fitting sets of data points extracted from Arabic letter contours.

A least-squares fitting approach is employed to determine optimal control points, ensuring an accurate approximation of the original shapes.

Modern research has demonstrated the effectiveness of least-squares Bézier fitting in handling complex datasets and irregular contours [4]. Moreover, trigonometric Bézier curves have recently been applied successfully in reconstructing Arabic calligraphy, showing superior adaptability to curved and decorative structures [5].

This research aims to demonstrate the effectiveness of the proposed approach in accurately reconstructing Arabic font outlines, while maintaining smoothness and visual fidelity.

The results highlight the potential of advanced basis functions in modern font design, digital typography, and shape modeling applications.

Definition 1

The proposed trigonometric-exponential basis with two shape parameters is defined as follows:

$$\begin{aligned}\widetilde{B_0^3}(t) &= \cos^2\left(\frac{\pi t}{2}\right) e^{-\alpha \sin\left(\frac{\pi t}{2}\right)} \\ \widetilde{B_1^3}(t) &= \sin\left(\frac{\pi t}{2}\right) \cos^2\left(\frac{\pi t}{2}\right) \\ \widetilde{B_2^3}(t) &= \cos\left(\frac{\pi t}{2}\right) \sin^2\left(\frac{\pi t}{2}\right) \\ \widetilde{B_3^3}(t) &= \sin^2\left(\frac{\pi t}{2}\right) e^{-\beta \cos\left(\frac{\pi t}{2}\right)},\end{aligned}$$

where $t \in [0, 1]$; $\alpha, \beta > 0$. Define the normalization function

$$W(t) = \sum_{i=0}^3 \widetilde{B_i^3}(t).$$

The final basis is

$$B_i^3(t) = \frac{\widetilde{B_i^3(t)}}{W(t)}, i = 0, 1, 2, 3. \quad (1)$$

Figure 1 illustrates the basis functions (1), for $\alpha = \beta = 0.5$.

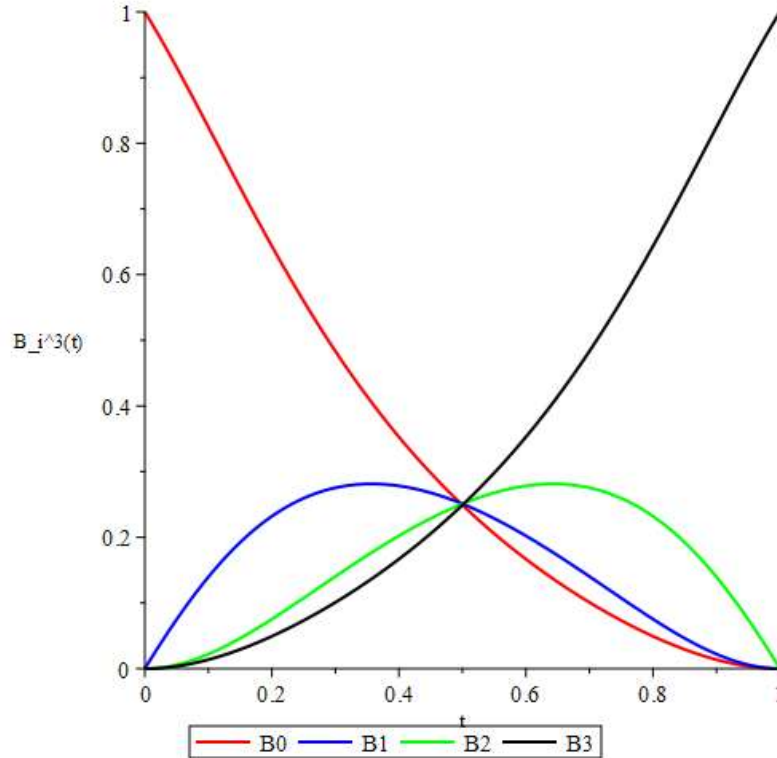


Figure 1

Theorem 1

The basis functions in (1) are non-negative.

Proof:

We have

$$B_0^3(t) = \frac{\widetilde{B_0^3(t)}}{W(t)} = \frac{\cos^2\left(\frac{\pi t}{2}\right)e^{-\alpha \sin\left(\frac{\pi t}{2}\right)}}{\widetilde{B_0^3(t)} + \widetilde{B_1^3(t)} + \widetilde{B_2^3(t)} + \widetilde{B_3^3(t)}},$$

for $t \in [0, 1]$ and $\alpha, \beta > 0$, the numerator of $B_0^3(t)$ is a product of two non-negative terms, and the sum in the denominator is positive. So that $B_0^3(t) \geq 0$. We can see with the same way that $B_i^3(t) \geq 0$ for $i = 1, 2, 3$.

Theorem 2

The basis functions in (1) satisfy $\sum_{i=0}^3 B_i^3(t) = 1$.

Proof:

We have

$$\sum_{i=0}^3 B_i^3(t) = \sum_{i=0}^3 \frac{\widetilde{B_i^3(t)}}{W(t)} = \frac{\widetilde{B_0^3(t)} + \widetilde{B_1^3(t)} + \widetilde{B_2^3(t)} + \widetilde{B_3^3(t)}}{\widetilde{B_0^3(t)} + \widetilde{B_1^3(t)} + \widetilde{B_2^3(t)} + \widetilde{B_3^3(t)}} = 1.$$

Theorem 3

$$B_i^3(t) = B_{3-i}^3(1-t); \quad \alpha = \beta, i = 0, 1, 2, 3.$$

Proof:

We know that

$$\cos\left(\frac{\pi t}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi t}{2}\right) = \sin\left(\frac{\pi}{2}(1-t)\right)$$

and

$$\sin\left(\frac{\pi t}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi t}{2}\right) = \cos\left(\frac{\pi}{2}(1-t)\right)$$

for $t \in [0, 1]$. Let's prove $B_0^3(t) = B_3^3(1-t)$.

If $\alpha = \beta$ we have

$$\begin{aligned} B_0^3(t) &= \frac{\widetilde{B_0^3(t)}}{W(t)} = \frac{\cos^2\left(\frac{\pi t}{2}\right)e^{-\alpha \sin\left(\frac{\pi t}{2}\right)}}{W(t)} \\ &= \frac{\sin^2\left(\frac{\pi}{2}(1-t)\right)e^{-\beta \cos\left(\frac{\pi}{2}(1-t)\right)}}{W(t)} = B_3^3(1-t). \end{aligned}$$

Now, let's prove $B_1^3(t) = B_2^3(1-t)$. We have

$$\begin{aligned} B_1^3(t) &= \frac{\widetilde{B_1^3(t)}}{W(t)} = \frac{\sin\left(\frac{\pi t}{2}\right)\cos^2\left(\frac{\pi t}{2}\right)}{W(t)} \\ &= \frac{\cos\left(\frac{\pi}{2}(1-t)\right)\sin^2\left(\frac{\pi}{2}(1-t)\right)}{W(t)} = B_2^3(1-t). \end{aligned}$$

Now, let's prove $B_2^3(t) = B_1^3(1-t)$. We have

$$\begin{aligned} B_2^3(t) &= \frac{\widetilde{B_2^3(t)}}{W(t)} = \frac{\cos\left(\frac{\pi t}{2}\right)\sin^2\left(\frac{\pi t}{2}\right)}{W(t)} \\ &= \frac{\sin\left(\frac{\pi}{2}(1-t)\right)\cos^2\left(\frac{\pi}{2}(1-t)\right)}{W(t)} = B_1^3(1-t). \end{aligned}$$

It is remained to prove $B_3^3(t) = B_0^3(1-t)$, We have

$$\begin{aligned} B_3^3(t) &= \frac{\widetilde{B_3^3(t)}}{W(t)} = \frac{\sin^2\left(\frac{\pi t}{2}\right)e^{-\beta \cos\left(\frac{\pi t}{2}\right)}}{W(t)} \\ &= \frac{\cos^2\left(\frac{\pi}{2}(1-t)\right)e^{-\alpha \sin\left(\frac{\pi}{2}(1-t)\right)}}{W(t)} = B_0^3(1-t). \end{aligned}$$

2. Bézier-Like Curves and the Meaning of Their Shape Parameters

A Bézier-like curve is a parametric curve defined as a linear combination of control points with respect to a set of basis functions that generalize the classical Bernstein basis while preserving its fundamental geometric properties. Formally, a Bézier-like curve of degree n is given by

$$P(t) = \sum_{i=0}^n P_i \varphi_i(t), \quad t \in [0, 1].$$

Where P_i are control points and $\varphi_i(t)$ are basis functions satisfying specific conditions [2,9].

In modern formulations, Bézier-like curves incorporate shape parameters into the basis functions:

$$P(t, \alpha, \beta) = \sum_{i=0}^n P_i \varphi_i(t, \alpha, \beta), \quad t \in [0, 1].$$

where α and β are adjustable parameters that control the shape of the curve without modifying the control points. Such parameterized constructions provide enhanced flexibility in geometric modeling and curve design [6]. Now let's study the influence of shape-parameters on the curve: Suppose we have four control points

$P_0 = (0, 0)$, $P_1 = (1, 2)$, $P_2 = (7, 2)$, and $P_3 = (9, 0)$. We will study the influence of different α and β on the curve

$$P(t) = \widetilde{B_0^3}P_0 + \widetilde{B_1^3}P_1 + \widetilde{B_2^3}P_2 + \widetilde{B_3^3}P_3, \quad t \in [0, 1]. \quad (2)$$

We have three cases:

Case1 ($\alpha \gg \beta$)

In this case, $B_0^3(t)$ becomes very small, so the influence of the control point P_0 is reduced. The curve departs quickly P_0 and is mainly governed by the control points P_1, P_2 and P_3 causing it to shift toward the later part of the control polygon. Figure 2 illustrates the curve $P(t)$ in (2) for $\alpha = 25$ and $\beta = 0.1$.

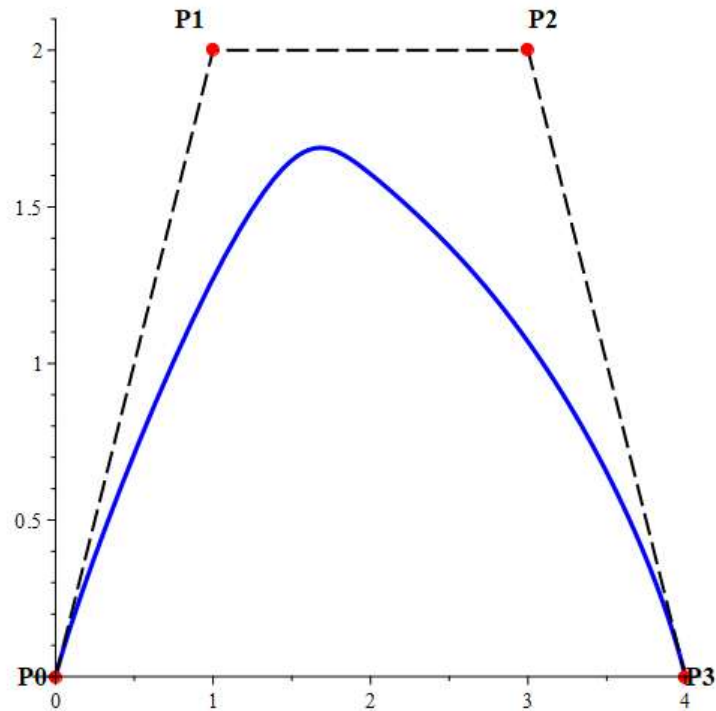


Figure 2

Case2 ($\alpha \ll \beta$)

In this case $B_3^3(t)$ becomes very small so the influence of P_3 is reduced and the curve approaches P_3 more slowly.

The curve approaches P_3 more slowly and is mainly influenced by P_0, P_1 and P_2 , shifting toward the initial part of the control polygon.

Figure 3 illustrates the curve $P(t)$ in (2) for $\alpha = 0.1$ and $\beta = 24$.

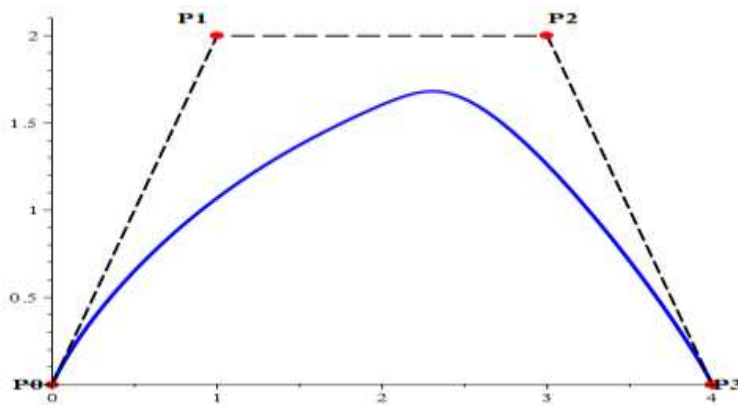


Figure 3

Case3 ($\alpha \approx \beta$)

In this case the influence of endpoints is balanced and there is no directional bias. Figure 4 illustrates the curve $P(t)$ in (2) for $\alpha = \beta = 2$.

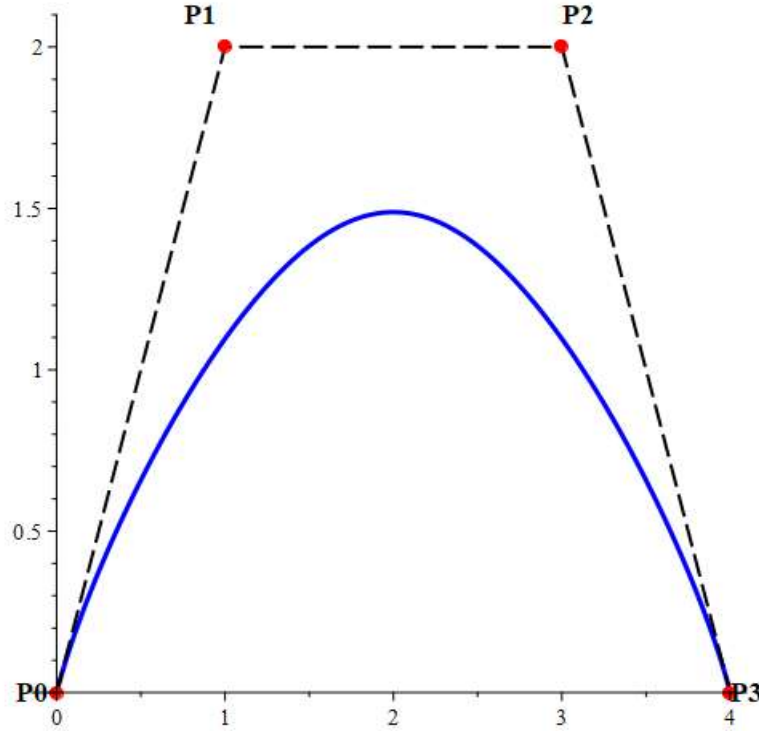


Figure 4

3- Curve Fitting Using the Proposed Basis

Bézier curve fitting is a technique in which a parametric curve defined by Bernstein basis functions is used to approximate a set of data points, where the control points are chosen to minimize the deviation between the curve and the data using least squares methods [7]. Suppose we are given the data points

$$Q_0 = (6.4, 5.6), Q_1 = (6.6, 5.2), Q_2 = (6.8, 4.2), Q_3 = (6.3, 3.2), Q_4 = (5.8, 2.8). \quad (3)$$

And we want to fit these data using cubic Bézier curve with the standard basis of Bernstein then computing the least squares sum and comparing it with the least squares sum obtained through using like- Bézier curve with the proposed trigonometric basis (1), for different values of α and β . Cubic Bézier curve is expressed as:

$$P(t) = \sum_{i=0}^3 P_i B_i^3(t)$$

where P_i are the control points and $B_i^3(t)$ are the basis functions. The first and last control points P_0 and P_3 are fixed, while the remaining control points are computed by minimizing the least-squares error sum between the curve and the given data points, which is a standard approach in modern Bézier curve fitting and geometric approximation methods [8,9].

Now we will construct a table to compute the least-squares sum using the standard basis, and then using the proposed basis for different values of the shape parameters.

Table 1

Value of shape- parameters	Least square sum with the proposed basis	Least square sum with Bernstein basis
$\alpha = 1, \beta = 0.1$	0.029607	0.028000
$\alpha = 1, \beta = 0.2$	0.028017	0.028000
$\alpha = 1, \beta = 0.3$	0.026603	0.028000
$\alpha = 1, \beta = 0.4$	0.025358	0.028000
$\alpha = 1, \beta = 0.5$	0.024272	0.028000
$\alpha = 1, \beta = 0.6$	0.023335	0.028000
$\alpha = 1, \beta = 0.7$	0.022539	0.028000
$\alpha = 1, \beta = 0.8$	0.021872	0.028000
$\alpha = 1, \beta = 0.9$	0.021326	0.028000
$\alpha = 1, \beta = 1$	0.020889	0.028000
$\alpha = 1, \beta = 10$	0.038887	0.028000
$\alpha = 1, \beta = 50$	0.040757	0.028000
$\alpha = 1, \beta = 150$	0.040757	0.028000
$\alpha = 1, \beta = 420$	0.040757	0.028000
$\alpha = 0.1, \beta = 1$	0.025678	0.028000
$\alpha = 0.2, \beta = 1$	0.024691	0.028000
$\alpha = 0.3, \beta = 1$	0.023834	0.028000
$\alpha = 0.4, \beta = 1$	0.023100	0.028000
$\alpha = 0.5, \beta = 1$	0.022483	0.028000
$\alpha = 0.6, \beta = 1$	0.021976	0.028000
$\alpha = 0.7, \beta = 1$	0.021571	0.028000
$\alpha = 0.8, \beta = 1$	0.021260	0.028000
$\alpha = 0.9, \beta = 1$	0.021035	0.028000
$\alpha = 2, \beta = 1$	0.022213	0.028000
$\alpha = 11, \beta = 2$	0.031082	0.028000
$\alpha = 20, \beta = 2$	0.031778	0.028000
$\alpha = 40, \beta = 2$	0.031802	0.028000
$\alpha = 121, \beta = 2$	0.031802	0.028000
$\alpha = 150, \beta = 2$	0.031802	0.028000
$\alpha = 300, \beta = 2$	0.031802	0.028000
$\alpha = 2, \beta = 2$	0.019578	0.028000
$\alpha = 3, \beta = 3$	0.023910	0.028000
$\alpha = 15, \beta = 15$	0.080387	0.028000
$\alpha = 30, \beta = 30$	0.081663	0.028000
$\alpha = 70, \beta = 70$	0.081667	0.028000
$\alpha = 300, \beta = 290$	0.081667	0.028000
$\alpha = 520, \beta = 550$	0.081667	0.028000
$\alpha = 700, \beta = 690$	0.081667	0.028000

Now we will plot the Bézier curve using the Bernstein basis (blue curve) and the Bézier-like curve using the proposed basis (green curve) for the prominent cases in Table 1.

Case 1

The shape-parameters are equal, and for example $\alpha = \beta = 1$. We see that the least-squares sum using the proposed basis is less than that obtained with the standard Bernstein basis; therefore, the approximation with proposed basis is better.

Figure 5 illustrates Bézier curve and Bézier-like curve for $\alpha = \beta = 1$.

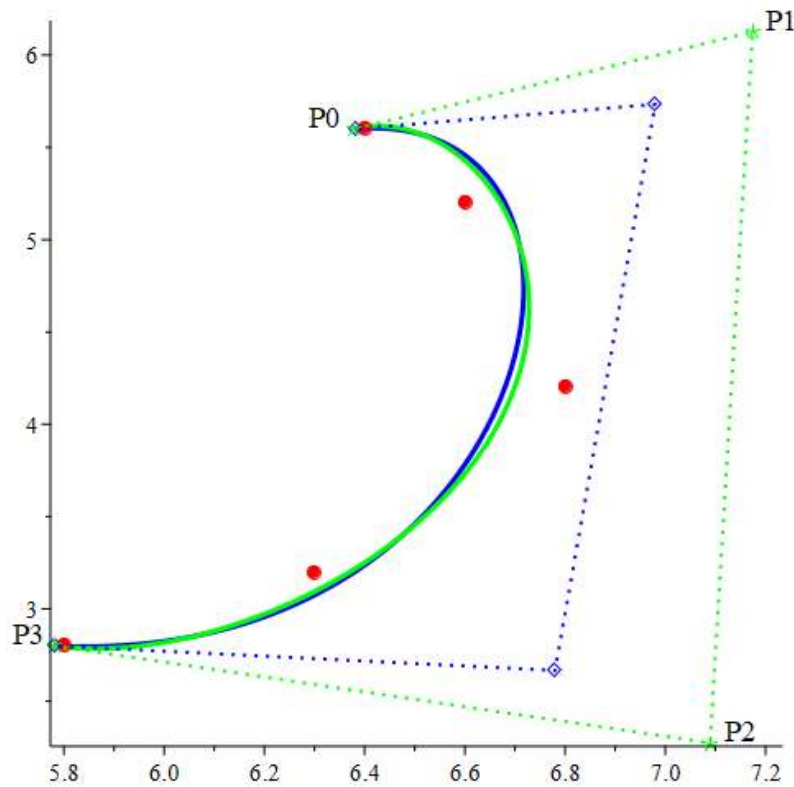


Figure 5

We observe that the Bézier-like curve is well-balanced and coincides with the Bézier curve at most points.

Case 2

The parameter β is much greater than α , for example let $\alpha = 1$, and $\beta = 420$. The Figure 6 shows both Bézier and Bézier-like curves.

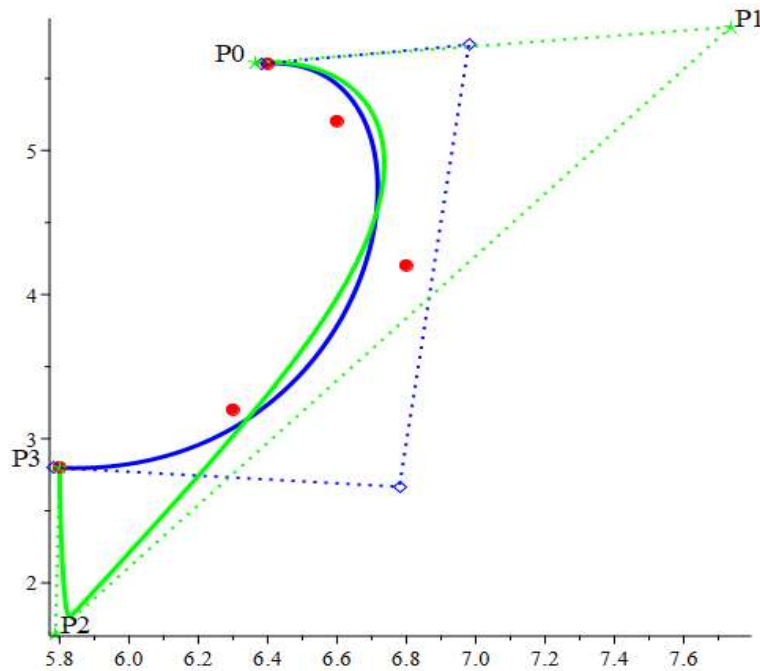


Figure 6

In this case, we observe that the control point P_3 has a weak influence on the curve due to the large value of β , which makes the basis function $B_3^3(t)$ small. So, the curve is strongly influenced by the last control point comes before P_3 , namely P_2 .

Case 3

The parameter α is much greater than β , for example let $\alpha = 300$, and $\beta = 2$. The Figure 7 shows both Bézier and Bézier-like curves.

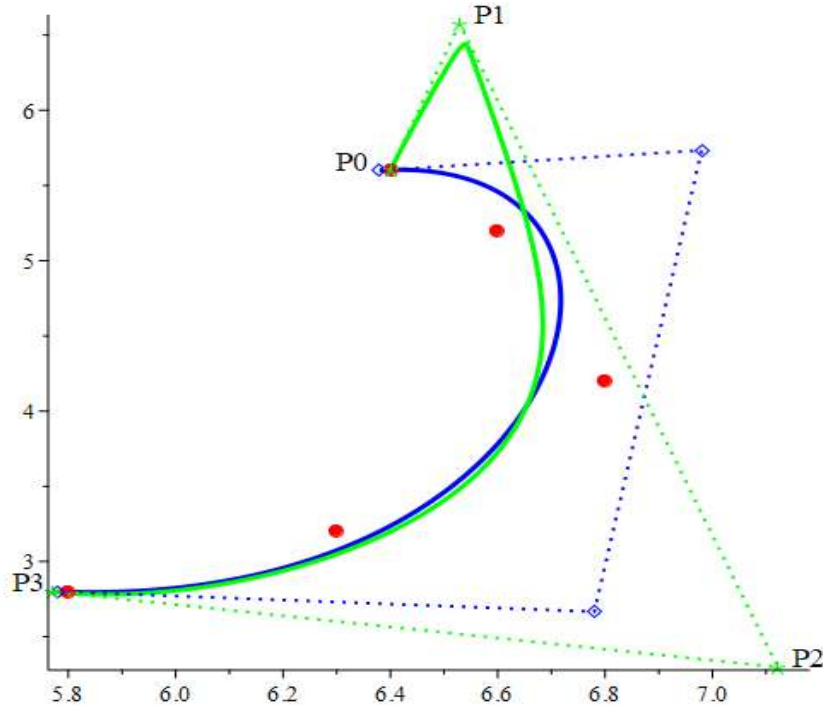


Figure 7

In this case, we observe that the control point P_0 has a weak influence on the curve due to the large value of α , which makes the basis function $B_0^3(t)$ small. So, the curve is strongly influenced by the control point comes directly after P_0 , namely P_1 .

Case 4

The shape-parameters are equal and close relatively to one, for example $\alpha = \beta = 2$. Figure 8 illustrates Bézier curve and Bézier-like curve for this case.

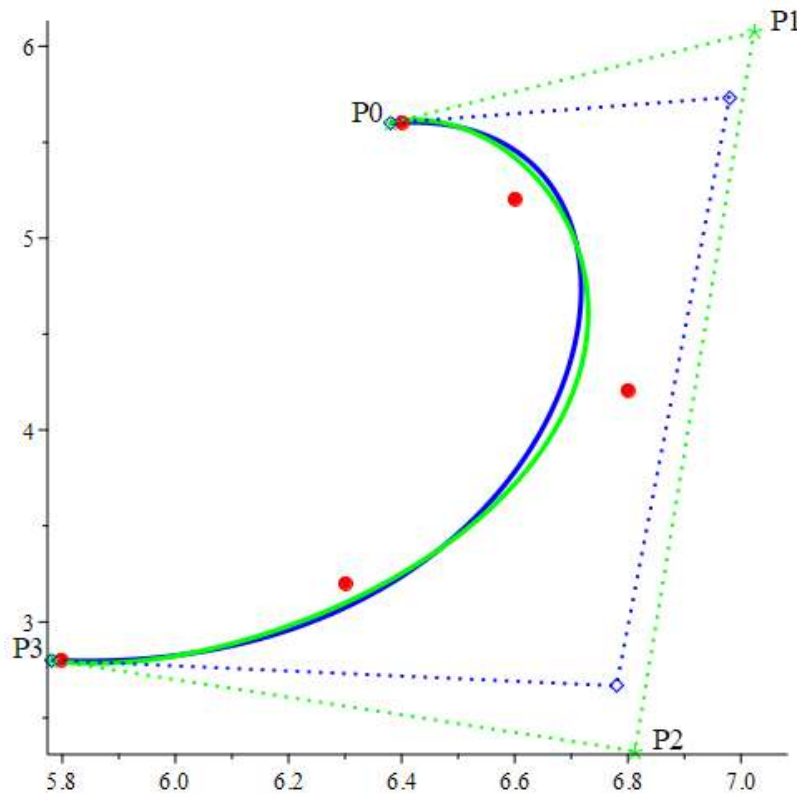


Figure 8

We observe from Table 1 that the least-squares sum in this case is 0.019578. This indicates that the best fit using the proposed basis occurs when the shape parameters α and β are both equal to the value 2.

Case 5

Both parameters are great and the difference between them is small, for example $\alpha = 700$, and $\beta = 690$, Figure 9 illustrates Bézier curve and Bézier-like curve in this case.

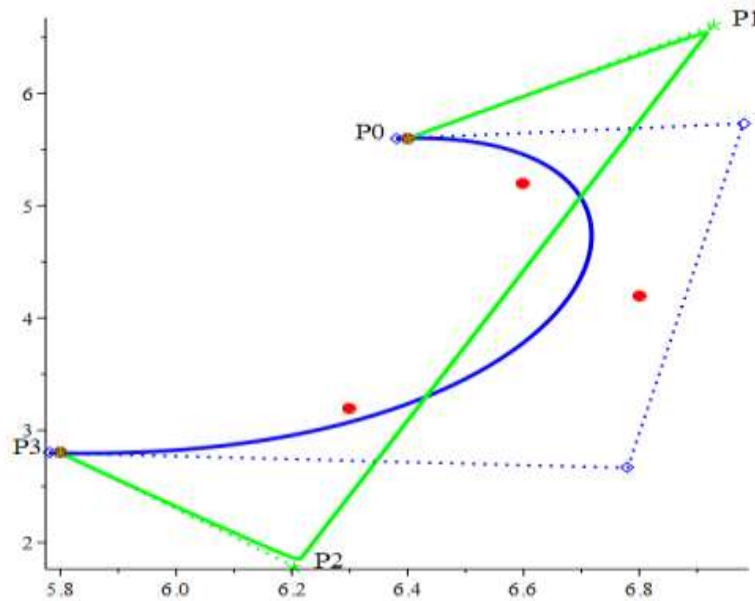


Figure 9

In this case, $B_0^3(t)$ is small due to the large value of α , and $B_3^3(t)$ is also small due to the large value of β . Therefore, the control points P_0 and P_3 have a weak influence on the curve. The main influence is exerted by the remaining control points P_1 and P_2 .

4- Applications

Digital font design, particularly for complex scripts such as Arabic, relies heavily on parametric curve representations, most notably Bézier curves [10,11]. In modern font development, each glyph is constructed using quadratic or cubic Bézier curves, which define smooth outlines through a set of control points. This approach allows designers to achieve high precision, scalability, and aesthetic consistency, as the curves can be continuously adjusted without loss of quality [12]. Bézier representations are resolution independent, meaning that the same glyph can be rendered clearly at any size or on any device. Moreover, the use of control points provides intuitive geometric manipulation, enabling fine-tuning of curvature, stroke contrast, and structural balance features that are essential in replicating the fluidity and calligraphic nature of scripts like Arabic [13].

In contrast, early font design methods were based on bitmap representations, where each character was stored as a grid of pixels. In this approach, the shape of a glyph is fixed at a specific resolution, and scaling the font often leads to distortion, jagged edges, and loss of detail [14]. Bitmap fonts lack flexibility, as each size or style must be designed separately, making them inefficient for modern multi-resolution environments. Additionally, pixel-based representations do not inherently support smooth curves, which limits their ability to accurately capture the continuous and flowing forms required in high-quality typography.

The fundamental difference between Bézier-based and bitmap-based font design lies in scalability, flexibility, and geometric precision. Bézier curves provide a mathematically continuous and scalable model, whereas bitmap techniques rely on discrete approximations. As a result, Bézier-based fonts have become the standard in contemporary digital typography, enabling efficient rendering, high-quality scaling, and faithful representation of complex scripts across diverse digital platforms [15].

Now we will use the proposed basis to fit the contour data of the Arabic letter “Baa” shown in Figure 10, then compare it with the fitting obtained using the Bernstein basis

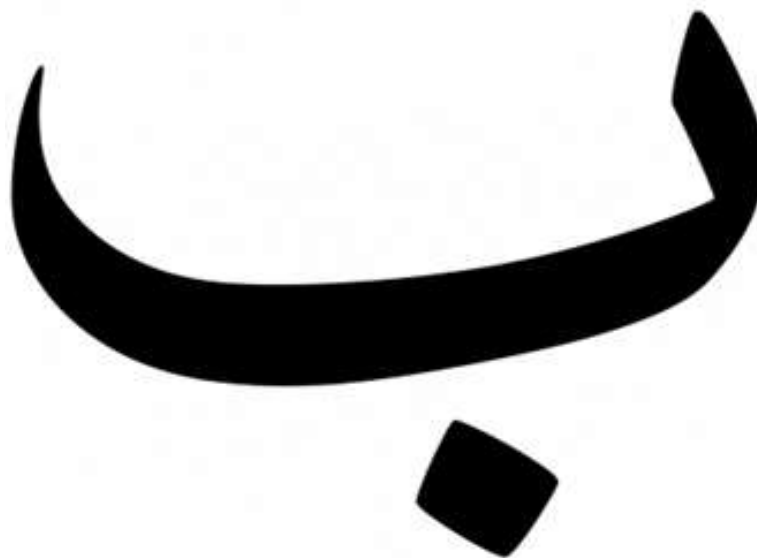


Figure 10

We will divide the contour of the letter into ten sets of data, and then fit every data alone using connected Bézier curves.

$$D_1 = \{(6.4, 5.6), (6.6, 5.2), (6.8, 4.2), (6.3, 3.2), (5.8, 2.8)\}$$

$$D_2 = \{(5.8, 2.8), (5, 2.5), (3, 2.2), (1.8, 2.3)\}$$

$$D_3 = \{(1.8, 2.3), (1, 3), (0.8, 4), (0.9, 5)\}$$

$$D_4 = \{(0.9, 5), (1, 4), (1.3, 3.3), (2.3, 2.9), (4, 3), (5, 3.2), (6.5, 3.9)\}$$

$$D_5 = \{(6.5, 3.9), (6.1, 4.9)\}$$

$$D_6 = \{(6.1, 4.9), (6.4, 5.6)\}$$

$$D_7 = \{(4.5, 1.7), (5.2, 1.3)\}$$

$$D_8 = \{(5.2, 1.3), (4.8, 0.6)\}$$

$$D_9 = \{(4.8, 0.6), (4.1, 1)\}$$

$$D_{10} = \{(4.1, 1), (4.5, 1.7)\}$$

Figure 11 illustrates the data points $D_1, D_2, \dots, D_{10}$ connected with straight dashed line segments in different colors.

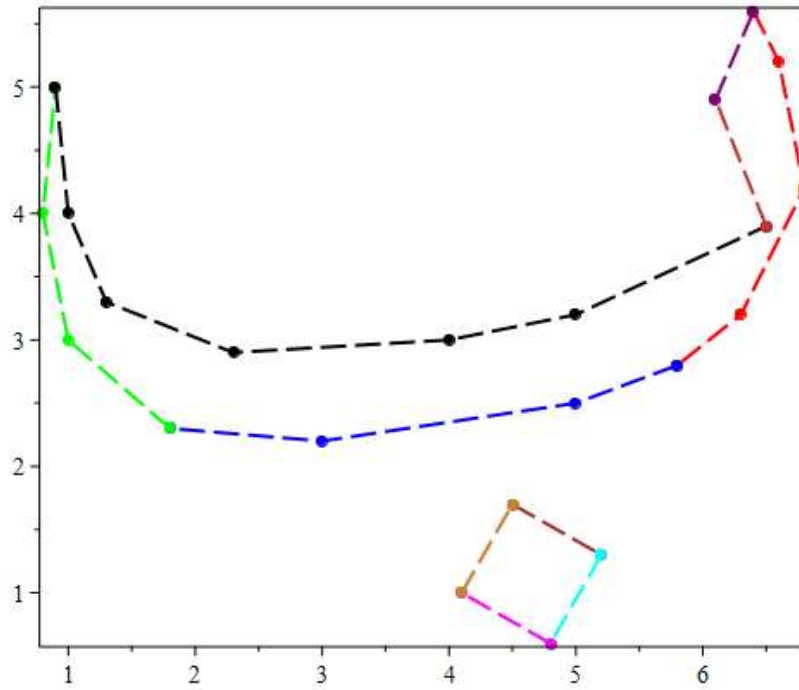


Figure 11

If we use cubic Bézier curve with Bernstein basis to fit the data $D_1, D_2, \dots, D_{10}$, we get the result shown in Figure 12.

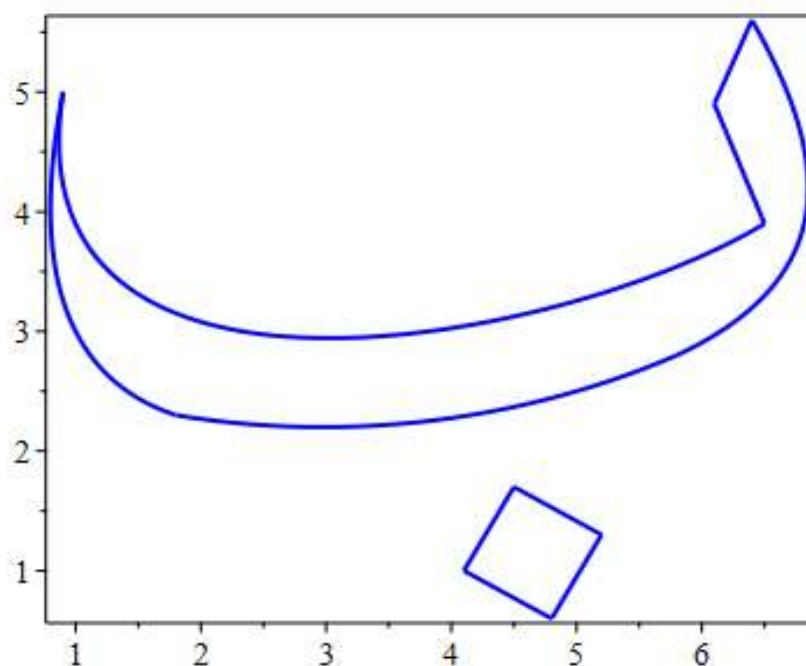


Figure 12

Now, fitting the data $D_1, D_2, \dots, D_{10}$, using Bézier-like curve with the proposed basis, for $\alpha = 0.1$ and $\beta = 0.4$, we get the result illustrated in Figure 13.

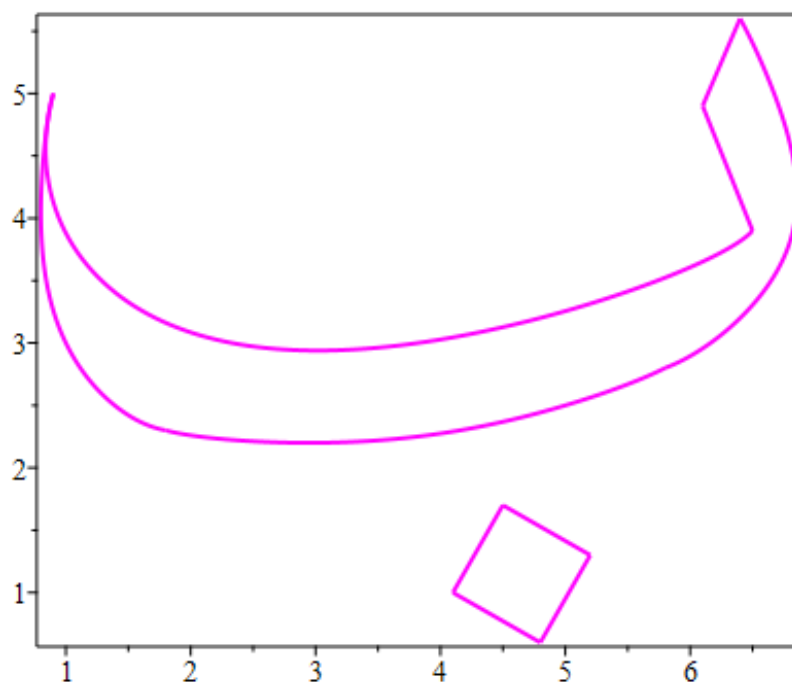


Figure 13

Now, fitting the data $D_1, D_2, \dots, D_{10}$, using Bézier-like curve with the proposed basis, for $\alpha = 0.2$ and $\beta = 0.1$, we get the result illustrated in Figure 14.

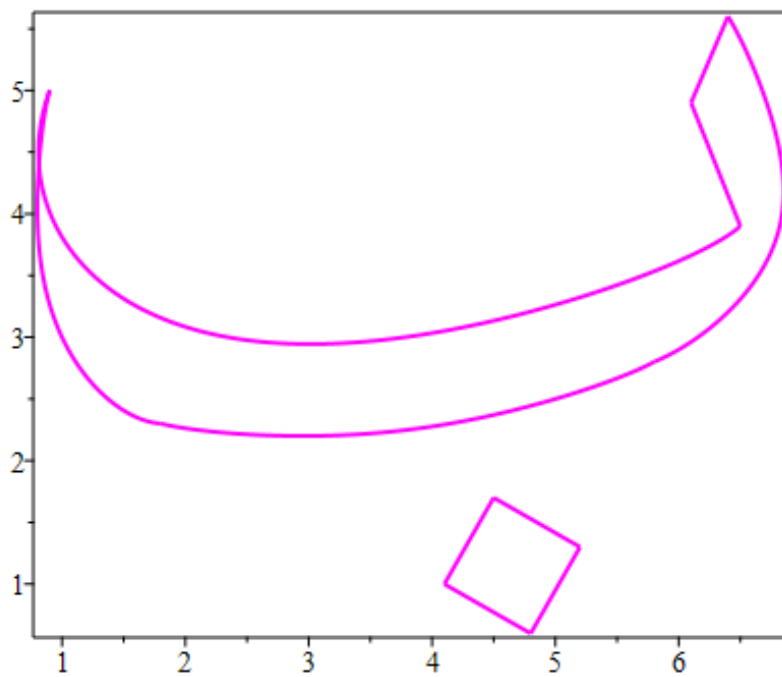


Figure 14

For big alpha and small beta; for example, $\alpha = 75$ and $\beta = 0.4$, we get unbalanced case, i.e. deformed letter. Figure 15 illustrates this case.

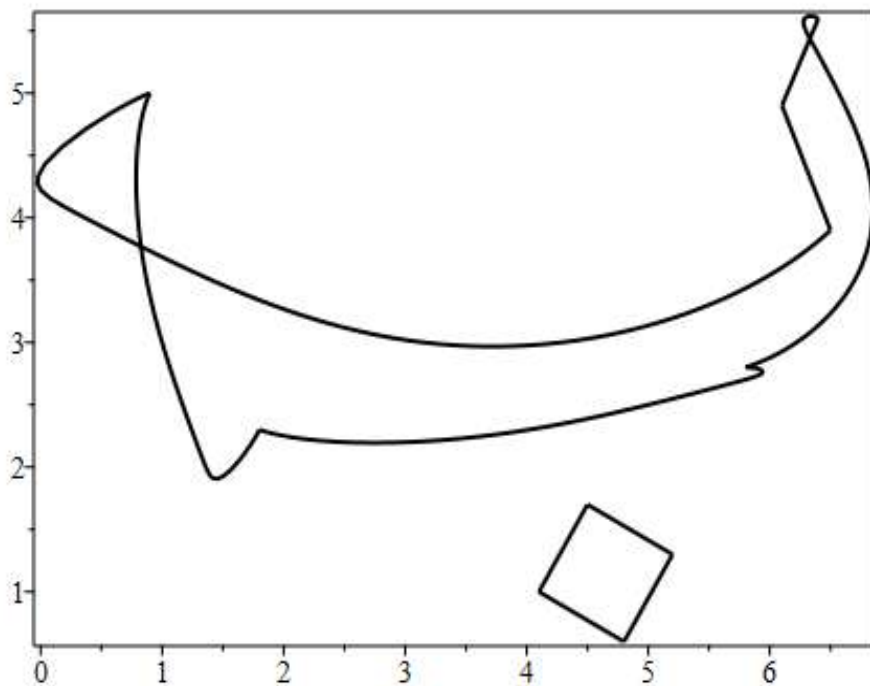


Figure 15

For small alpha and big beta; for example, $\alpha = 0.2$ and $\beta = 124$ we get unbalanced case too, i.e. deformed letter. Figure 16 illustrates this case.

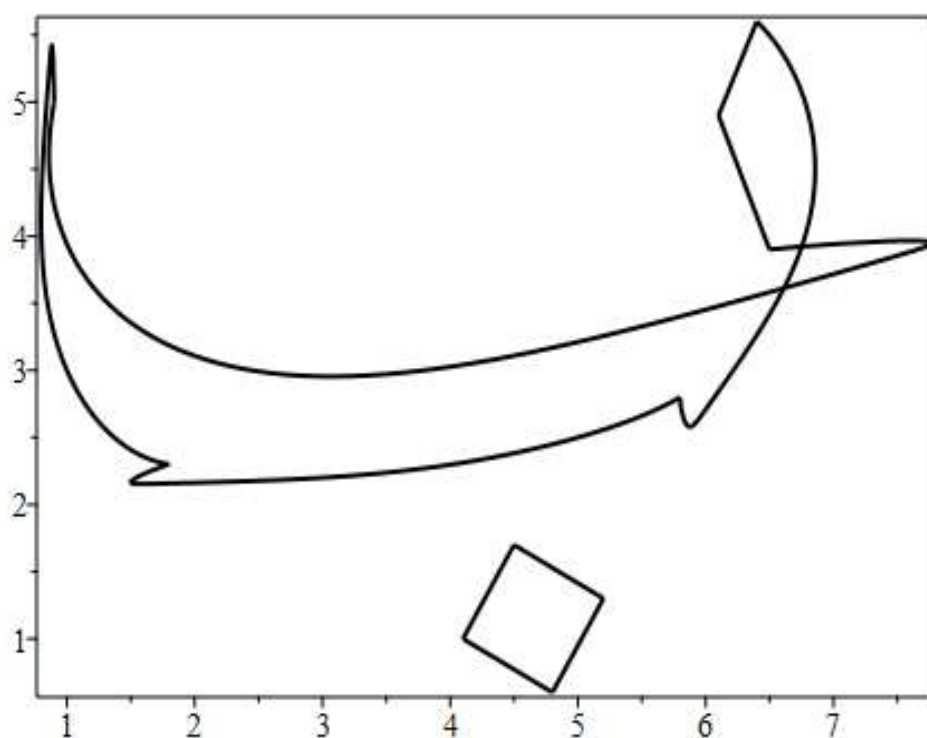


Figure 16

For big α and big β ; for example, $\alpha = 75$ and $\beta = 70$ we get unbalanced case, i.e. deformed letter. Figure 17 illustrates this case.

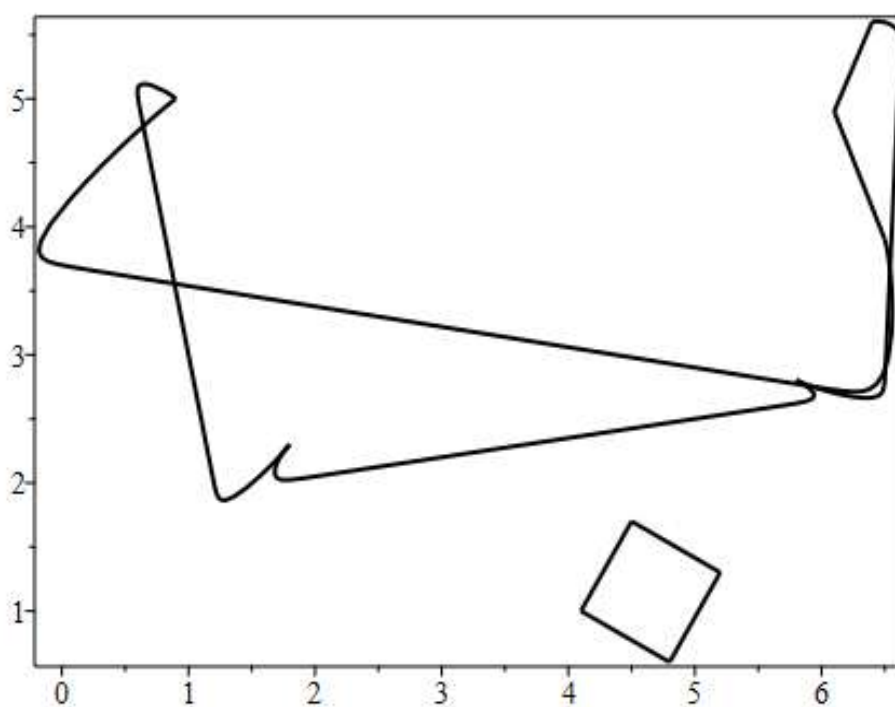


Figure 17

5. Results:

1. The proposed trigonometric–exponential basis preserves the essential geometric properties of the classical Bernstein basis, including non-negativity, partition of unity, endpoint interpolation, and convex hull preservation.
2. Introducing two shape parameters provides additional flexibility for controlling the geometric behavior of Bézier-like curves without increasing the polynomial degree or the number of control points.
3. When the parameter $\alpha \gg \beta$, the influence of the control point P_1 becomes weak, causing the curve to shift toward the later part of the control polygon.
4. When the parameter $\beta \gg \alpha$, the influence of the control point P_2 decreases significantly, causing the curve to move toward the initial part of the control polygon.
5. Numerical least-squares fitting experiments show that the proposed basis achieves smaller fitting errors than the classical Bernstein basis for several parameter choices.
6. The proposed Bézier-like curves successfully reconstruct the contour of the Arabic letter “Baa” with high smoothness and better preservation of calligraphic features compared with the standard cubic Bézier representation.
7. The proposed basis maintains curve smoothness and geometric continuity while allowing significant shape variation through small changes in the parameters α and β .
8. The fitting process demonstrates that the proposed Bézier-like curves can approximate complex Arabic letter contours using fewer geometric modifications compared with classical Bézier curves.
9. The trigonometric–exponential formulation enhances the adaptability of the curve near highly curved regions, making it suitable for modeling decorative and calligraphic structures in Arabic typography.
10. The proposed method provides a scalable and resolution-independent representation of Arabic glyphs, which is advantageous for modern digital font rendering systems and computer-aided design applications.

6. Conclusions:

In this research, a new trigonometric–exponential Bézier-like basis with two shape parameters was introduced and applied to Arabic font fitting. The proposed basis preserves the fundamental geometric properties required for geometric design while providing additional flexibility through controllable shape parameters. Theoretical analysis confirmed the positivity, partition of unity, and endpoint interpolation properties of the basis functions.

The geometric influence of the shape parameters was studied through several numerical examples, showing that the proposed model enables effective local shape adjustment without modifying the control polygon structure. Least-squares fitting experiments demonstrated that the proposed Bézier-like curves can achieve better approximation accuracy than classical cubic Bézier curves for suitable parameter values.

The application to the Arabic letter “Baa” showed that the proposed approach is capable of reconstructing Arabic glyph contours smoothly and accurately while preserving the aesthetic characteristics of Arabic calligraphy. Therefore, the proposed method can serve as an effective tool in digital typography, computer-aided geometric design, Arabic font generation, and curve modeling applications.

Future work may include extending the proposed basis to Bézier surfaces, developing adaptive parameter selection techniques, and applying the method to complete Arabic font families and real-time font rendering systems.

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